SPH3U: The BIG Five

Last class we found three equations to help describe motion with constant acceleration. A bit more work along those lines would allow us to find two more equations which give us a complete set of equations for the five kinematic quantities.

A: The BIG Five – Revealed!

Here are the BIG five equations for uniformly accelerated motion.

The **BIG** Five	$\vec{v}_1$	$\vec{v}_2$	$\Delta\vec{x}$	$\vec{a}$	Δt
$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$	✓	✓		✓	✓
$\Delta\vec{x} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$	✓		✓	✓	✓
$\Delta\vec{x} = \vec{v}_2\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$		✓	✓	✓	✓
$\Delta\vec{x} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$	✓	✓	✓		✓
$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{x}$	✓	✓	✓	✓	

1. **Describe.** Define carefully each of the kinematic quantities in the chart below.

$\vec{v}_1$	initial velocity
$\vec{v}_2$	final velocity
$\Delta\vec{x}$	change in position (displacement)
$\vec{a}$	acceleration
Δt	change in time

2. **Reason.** What condition must hold true (we mentioned these in the previous investigation) in order to use the big 5 kinematic equations? acceleration must be constant

B: As Easy as 3-4-5

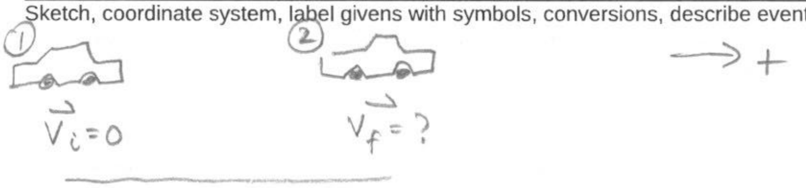
Solving a problem involving uniformly accelerated motion is as easy as 3-4-5. As soon as you know **three** quantities, you can always find a **fourth** using a BIG **five**! Write your solutions carefully using our solution process. Note that we are focusing on certain steps in our work here – in your homework, make sure you complete all the steps!

Problem 1

A traffic light turns green and an anxious student floors the gas pedal, causing the car to acceleration at 3.4 m/s^2 for a total of 10.0 seconds. We wonder: How far did the car travel in that time and what's the big rush anyways?

A: Pictorial Representation

Sketch, coordinate system, label givens with symbols, conversions, describe events



$\vec{a} = 3.4\ m/s^2$

$\Delta x = ?$

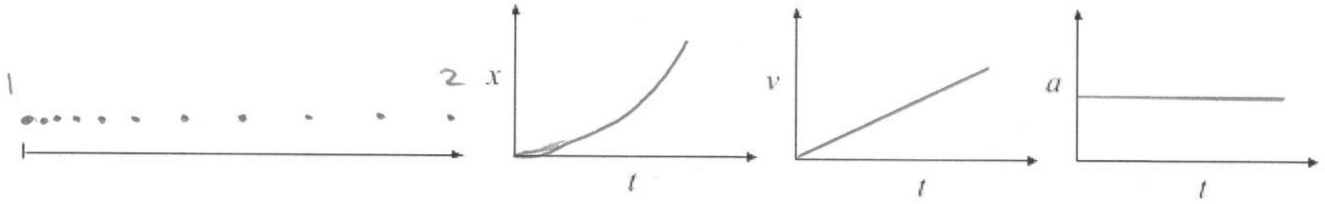
$\Delta t = 10\ s$

Emmy says, "I am given only two numbers, the acceleration and time. I need three to solve the problem. I'm stuck!" Explain how to help Emmy.

↳ stopped at start, so $\vec{v}_i = 0$

C: Physics Representation

Motion diagram, motion graphs, velocity vectors, events



D: Mathematical Representation

Describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{v_i} = 0$

$v_f = ?$

$\Delta\vec{x} = ?$

$\vec{a} = 3.4\ m/s^2$

$\Delta t = 10.0\ s$

① find final velocity

$\vec{v_2} = \vec{v_1} + \vec{a}\Delta t$

$\vec{v_2} = 0 + (3.4)(10.0)$

$\vec{v_2} = 34.0\ m/s$

② find distance travelled

$\Delta x = \vec{v}_1 t + \frac{1}{2}\vec{a}(\Delta t)^2$

$\Delta x = (0)(10) + \frac{1}{2}(3.4)(10)^2$

$\Delta x = 170\ m$

∴ travelled 170 m

C: You've Got Problems: Complete these problems on a separate solution sheet

(1) **Crash Test.** An automobile safety laboratory performs crash tests of vehicles to ensure their safety in high-speed collisions. The engineers set up a head-on crash test for a Smart Car which collides with a solid barrier. The engineers know the car initially travels south at 100. km/h and the car crumples 0.780 m during the collision. The engineers have a couple of questions: How much time does the collision take? What was the car's acceleration during the collision? (0.0562 s, 495 m/s^2 [backwards])

(2) **Off the Wall.** An important part of Penny's swim race is when she turns around while pushing on the swimming pool wall. When she makes contact with the wall, she is travelling at 1.66 m/s east. After pushing against the wall for 0.30 s, she leaves contact with it and is travelling at 1.98 m/s west. What is her acceleration during this time? (12.1 m/s^2 west)

(3) **The Track.** A cart is placed at the bottom of an inclined track. It uses a spring to launch itself up the incline with a speed of 0.79 m/s. While travelling up and down the incline, the cart has an acceleration of 0.54 m/s^2. How much time does it take to make the complete trip up and back down to its starting position? (Hint: this is a one step problem) (2.9 s)

(4) **Taking Off.** A jumbo jet must reach a speed of 360 km/h on the runway for takeoff. What is the smallest constant acceleration needed to takeoff from a 1.80 km runway? Give your answer in m/s^2 (2.8 m/s^2)

(5) **Shuffleboard Disk.** A shuffleboard disk is accelerated at a constant rate from rest to a speed of 6.0 m/s over a 1.8 m distance by a player using a stick. The disk then loses contact with the stick and slows at a constant rate of 2.5 m/s^2 until it stops. What total distance does the disk travel? (Hint: how many events are there in this problem?) (9.0 m)

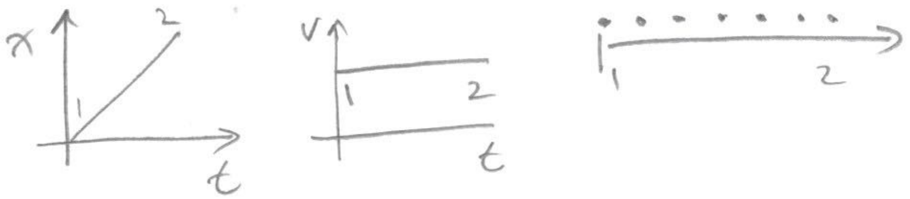
35. $\vec{v}_i = 30$ km/h 30 km/h $= 8.\bar{3}$ m/s $\rightarrow +$

$\vec{v}_f = 30$ km/h

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$$

$$8.3 = \frac{\Delta \vec{x}}{0.5}$$

$$\Delta \vec{x} = 4.2 \text{ m}$$



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37. $\vec{v}_i = 14.0$ m/s

$\vec{v}_f = 16.0$ m/s

$\Delta t = 4.80$ s

~~$a = ?$~~

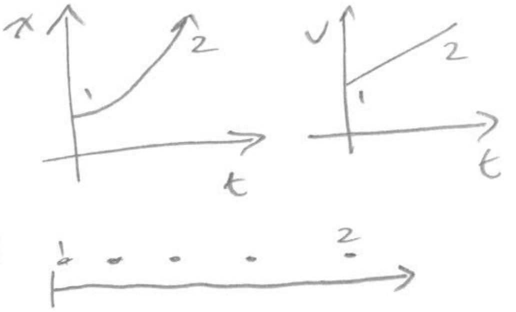
$\Delta x = ?$

$$\vec{x} = \tfrac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$$

$$\vec{x} = \tfrac{1}{2}(14.0 + 16.0)(4.8 \text{ s})$$

$$= \tfrac{1}{2}(30 \text{ m/s})(4.8 \text{ s})$$

$$\vec{x} = 72.0 \text{ m}$$



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43. $\vec{v}_i = ?$

$\vec{v}_f = 15.0$ m/s

$\Delta \vec{x} = 120$ m

$\Delta t = 5.60$ s

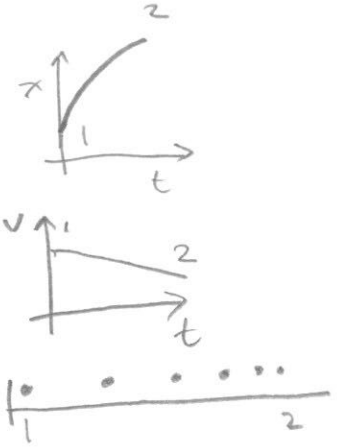
$$\Delta \vec{x} = \tfrac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$$

$$\frac{120 \text{ m}}{5.6} = \frac{\tfrac{1}{2}(\vec{V}_1 + 15.0 \text{ m/s})(5.6 \text{ s})}{5.6}$$

$$21.43 \text{ m/s} = \tfrac{1}{2}(\vec{V}_1 + 15.0 \text{ m/s})$$

$$42.86 \text{ m/s} = \vec{V}_1 + 15.0 \text{ m/s}$$

$$\boxed{27.9 \text{ m/s} = \vec{V}_1 \quad \text{slowing down}}$$



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44. $\vec{V}_1 = 0$ m/s

$\vec{V}_2 = 10.2$ m/s

$\vec{a} = ?$

$\Delta t = 2.5$ s

~~Δx~~

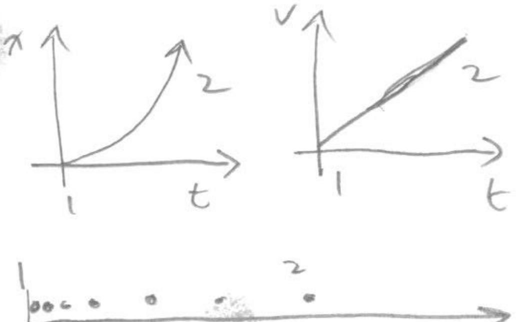
$$\vec{V}_2 = \vec{V}_1 + \vec{a}\Delta t$$

$$10.2 \tfrac{\text{m}}{\text{s}} = 0 + \vec{a}(2.5 \text{ s})$$

$$10.2 \tfrac{\text{m}}{\text{s}} = \vec{a}(2.5 \text{ s})$$

$$\vec{a} = \frac{10.2}{2.5}\frac{\text{m/s}}{\text{s}}$$

$$\boxed{\vec{a} = 4.1 \text{ m/s}^2}$$



(45) $\vec{v}_1 = 0$ m/s

$\vec{a} = 2.2$ m/s²

$\Delta t = 2.5$ s

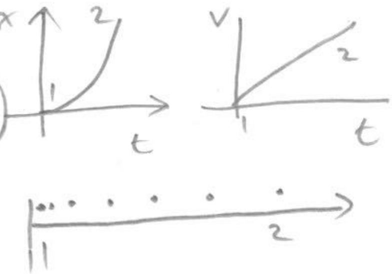
$\vec{v}_f = ?$

~~$\Delta\vec{x}$~~

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$\vec{v}_2 = 0 + (2.2 \frac{m}{s^2})(2.5 s)$

$\vec{v}_2 = 5.5$ m/s



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(46) $\vec{v}_1 = 13.0$ m/s

$V_2 = 0$ m/s

$\Delta t = 0.080$ s

$\vec{a} = ?$

~~$\Delta\vec{x}$~~

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

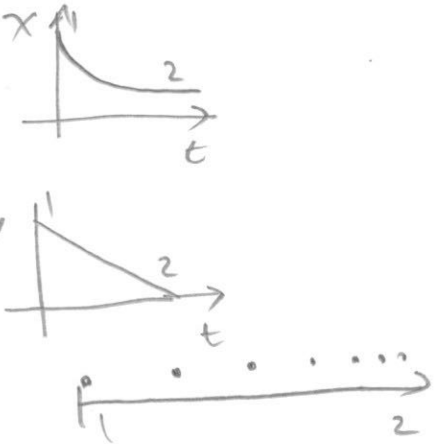
$0 = 13.0 m/s + \vec{a}(0.080 s)$

$-13.0 m/s \quad -13.0 m/s$

$\frac{-13.0 m/s}{0.080 s} = \frac{\vec{a}(0.080 s)}{0.080 s}$

$\boxed{-162.5 \, m/s^2 = \vec{a}}$

$\vec{a} = -160$ m/s²



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(56) $\vec{v}_i = 40$ km/h $= 11.1$ m/s

~~$\vec{v}_f$~~ $= ?$

$\Delta\vec{x} = ?$

$\vec{a} = 2.3$ m/s²

$\Delta t = 2.7$ s

$\Delta\vec{x} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$

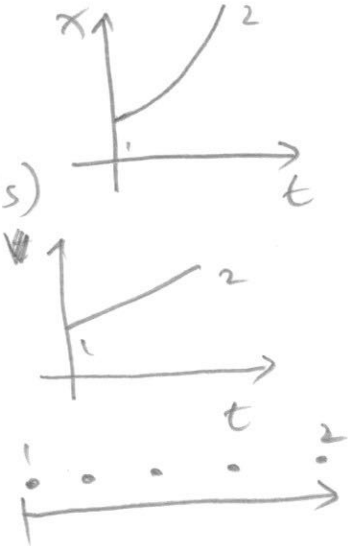
$\Delta\vec{x} = (11.1 \frac{m}{s})(2.7s) + \frac{1}{2}(2.3 \frac{m}{s^2})(2.7s)$

$\boxed{\Delta\vec{x} = 38.4 \, m}$

$\vec{V}_2 = \vec{V}_1 + \vec{a}\Delta t$

$\vec{V}_2 = 11.1 m/s + (2.3 \frac{m}{s^2})(2.7s)$

$\boxed{\vec{V}_2 = 17.3 \, m/s}$



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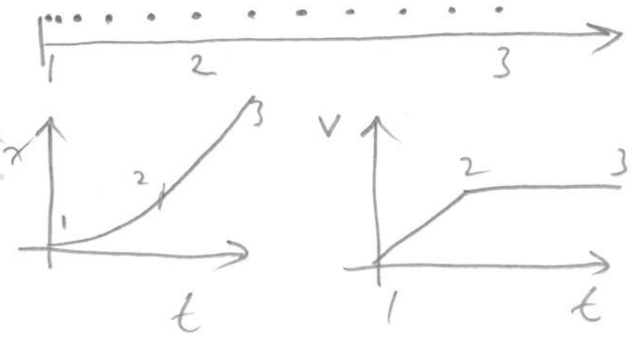
(58) Event 1 → start

Event 2 → finished acceleration

Event 3 → finish race



During acceleration (1→2)

$\vec{v}_i = 0$ m/s

$\vec{v}_f = ?$

$\Delta t = 3.5$ s

$\vec{a} = 2.8$ m/s²

$\Delta\vec{x} = ?$

$$\vec{x} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$= (0 \tfrac{m}{s})(3.5 s) + \tfrac{1}{2}(2.8 \tfrac{m}{s^2})(3.5 s)^2$$

$$\boxed{\vec{x} = 17.15 \text{ m}}$$

Find $\vec{v}_f$

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

$$\vec{v}_2 = 0 + (2.8 \tfrac{m}{s^2})(3.5 s)$$

$$\boxed{\vec{v}_2 = 9.8 \text{ m/s}}$$

Distance remaining = 100 − 17.2 m = 82.8 m

$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$$

$$9.8 \tfrac{m}{s} = \frac{82.8 m}{\Delta t}$$

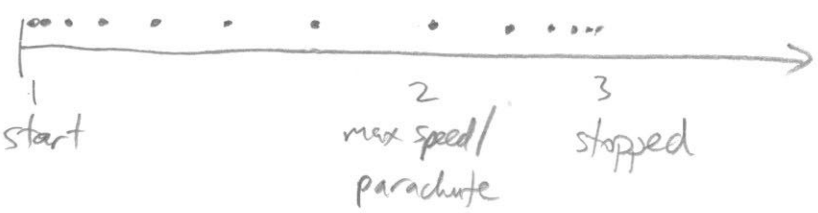
$$\Delta t = \frac{82.8 m}{9.8 m/s}$$

$$\Delta t = 8.4 s$$

$$\therefore \Delta t_{total} = 3.5 + 8.4$$

$$\boxed{\Delta t_{total} = 11.9 \text{ s}}$$

(65)



Between events 1-2

$\vec{V}_i = 0$

$\Delta\vec{x} = 450$ m

$\vec{a} = 14$ m/s^2

$\vec{V}_f = ?$

$\Delta t = ?$

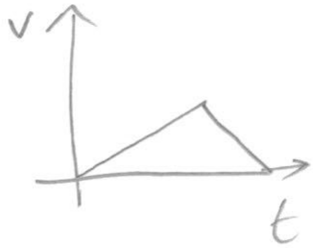
Find $\vec{V}_2$

$$\vec{V}_2^{\,2} = \vec{V}_1^{\,2} + 2\vec{a}\Delta\vec{x}$$

$$\vec{V}_2^{\,2} = (0)^2 + 2(14 \tfrac{m}{s^2})(450\text{ m})$$

$$\vec{V}_2^{\,2} = 12600 \tfrac{m^2}{s^2}$$

$$\vec{V}_2 = 112.2 \text{ m/s}$$



Now events 2-3

$\vec{V}_i = 112.2$ m/s

$\vec{V}_f = 0$ m/s

$\vec{a} = -7.0$ m/s^2

$\Delta\vec{x} = ?$

~~Δt~~

$$\vec{V}_F^{\,2} = \vec{V}_i^{\,2} + 2\vec{a}\Delta\vec{x}$$

$$0 = (112.2 \tfrac{m}{s})^2 + 2(-7.0 \tfrac{m}{s^2})\Delta x$$

$$0 = 12600 \tfrac{m^2}{s^2} - 14.0 \tfrac{m}{s^2}(\Delta x)$$

$$\frac{(14.0 \tfrac{m}{s^2})(\Delta x)}{14.0 \tfrac{m}{s^2}} = \frac{12600 \tfrac{m^2}{s^2}}{14.0 \text{ m/s}^2}$$

$$\Delta x = 900 \text{ m}$$

$\therefore$ total distance is $900 + 450 = 1350$ m

(rounds to 1400 m)