SPH3U: Calculating Acceleration

Defining Acceleration

The expression $\Delta\vec{v}/\Delta t$ represents the *change* in velocity occurring in each unit of time and is called *acceleration* $\vec{a}$:

$$\vec{a} = \frac{\vec{\Delta v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \qquad \vec{v} = \frac{\vec{\Delta x}}{t}$$

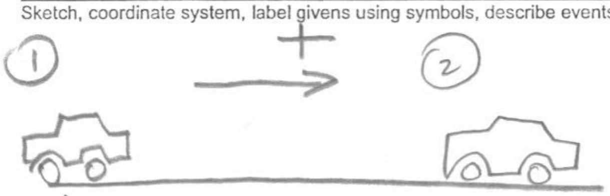
Note in the equation above, we wrote v_f and v_i for the final and initial velocities during some interval of time. If your time interval is defined by events 2 and 3, then v_3 is the final velocity and v_2 is the initial velocity.

B: Problem Solving

1. **Hit the Gas!** You are driving along the 401 and want to pass a large truck. You floor the gas pedal and begin to speed up. You start at 102 km/h, accelerate at a steady rate of 2.9 (km/h)/s (obviously not a sports car). What is your velocity after 5.3 seconds when you finally pass the truck?

A: Pictorial Representation

Sketch, coordinate system, label givens using symbols, describe events



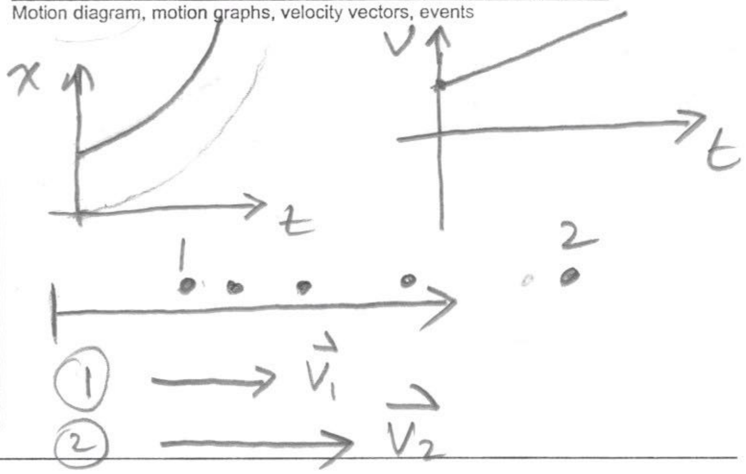
$\vec{v}_i = 102$ km/h $\vec{v}_f = ?$

$\vec{a} = 2.9$ km/h/s

$\Delta t = 5.3$ s

C: Physics Representation

Motion diagram, motion graphs, velocity vectors, events



B: Word Representation

Describe motion (no numbers), explain why, assumptions

– constant acceleration

D: Mathematical Representation

Number and describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{a}$ ✓
$\vec{v}_1$ ✓
v_2 ?
Δt ✓

$$\Delta t \times \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \times \Delta t$$

$$\vec{a}\Delta t = \vec{v}_2 - \vec{v}_1$$

$$+\vec{v}_1 \qquad +\vec{v}_1$$

$$\vec{a}\Delta t + \vec{v}_1 = \vec{v}_2$$

$$(2.9\ \text{km/h/s})(5.3\ \text{s}) + 102\ \text{km/h} = \vec{v}_2$$

$$15.37\ \text{km/h} + 102\ \text{km/h} = \vec{v}_2$$

$\vec{v}_2 = 117.37$ km

$\vec{v}_2 \doteq 117$ km/h

2. **The Rocket** A rocket is travelling upwards. A second engine begins to fire causing it to speed up at a rate of 21 m/s^2. After 4.3 seconds it reaches a velocity of 115 m/s and the engine turns off. What was the velocity of the rocket when the second engine began to fire?

To describe motion in the vertical direction, use the symbol *y* for the *vertical* position. All other symbols remain the same. In physics, the symbol *x* will only be used for *horizontal* position. The sketch for the situation should show the vertical motion and the coordinate system should show which vertical direction is the +*y*-direction. The motion diagram and the velocity vectors should point vertically.

A: Pictorial Representation

Sketch, coordinate system, label givens & unknowns using symbols, conversions, describe events

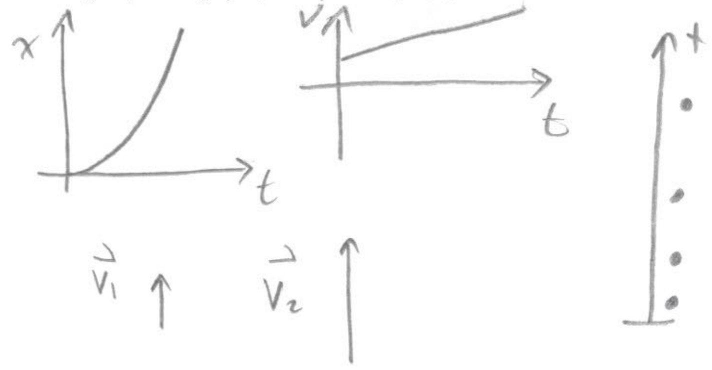
(2) $\vec{v}_2 = 115$ m/s ↑+

$\vec{a} = 21$ m/s^2

(1) $\vec{v}_1 = ?$ $\Delta t = 4.3$ s

C: Physics Representation

Motion diagram, motion graphs, velocity vectors, events



B: Word Representation

Describe motion (no numbers), explain why, assumptions

constant acceleration upwards (+ direction)

D: Mathematical Representation

Number and describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a}\Delta t = \vec{v}_f - \vec{v}_i$$

$$\vec{v}_i = \vec{v}_f - \vec{a}\Delta t$$

$$\vec{v}_i = 115 \text{ m/s} - (21 \tfrac{\text{m}}{\text{s}^2})(4.3 \text{ s})$$

$$= 115 \text{ m/s} - 90.3 \text{ m/s}$$

$$\vec{v}_i = 24.7 \text{ m/s}$$

$$\vec{v}_i \doteq 25 \text{ m/s [up]}$$

Calculating Acceleration Homework *(from Irwin Physics 11 p67)* **NOTE**: *NEED TO DO THESE ON A SEPARATE PIECE OF PAPER. INCLUDE PICTORIAL, PHYSICS, WORD AND MATHEMATICAL REPRESENTATIONS*

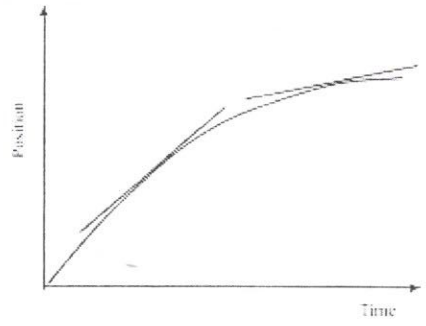
47. A pitcher can throw a baseball at 100 km/h. If the ball starts from rest and accelerates over 1.5 s, what is the acceleration of the ball? ANS: *19 m/s^2 [fwd]*	**48.** A car moving at 10.0 m/s north ends up moving 10.0 m/s south after a period of 12 s. What is its acceleration? ANS: *48. 1.7 m/s^2 [S]*	**52.** What is an object's final velocity if it accelerates at -2.0 m/s^2 for 2.3 s from a velocity of 50.0 km/h? ANS: *52. 9.3 m/s [fwd]*

SPH3U: Speeding Up or Slowing Down?

Here is one mystery concerning acceleration remaining to be solved. Our definition of acceleration, $\Delta v/\Delta t$, allows the result to be either positive or negative, but what does that mean? Today we will get to the bottom of this.

A: Acceleration in Graphs

Suppose the motion of a student is shown in the position-time graph to the right.



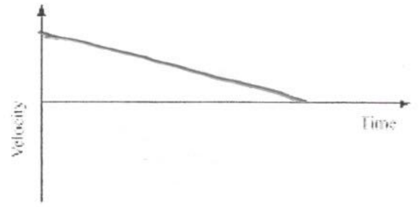
1. **Interpret.** Describe the motion of the student shown in the diagram.

travelling in + direction, quickly at first and then slowing down

2. **Interpret.** What does the slope of a tangent to a position-time graph represent?

Velocity

3. **Reason.** What is happening to the slope of the tangent in the position-time graph as we move from left to right along the curve? What does this tell us about the velocity?



slope is decreasing
↳ velocity is decreasing

4. **Predict.** What will the velocity-time graph look like? Draw a line or curve on the velocity time graph to the right..

5. **Interpret.** Is the cart speeding up or slowing down? Use the two tangents to the graph to help explain.

Slowing down – steepness of x-t graph is decreasing

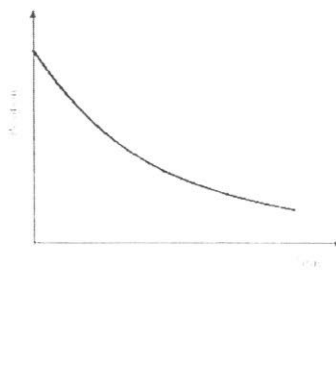
> To help interpret position graphs, we will use the *tangent trick*. Use a ruler or pencil as the tangent line to a position graph. Interpret the slope of the tangent. Then move the tangent to a new spot along the graph and interpret. Decide if the object is speeding up or slowing down. This trick can also be used to decide how to sketch a position graph.

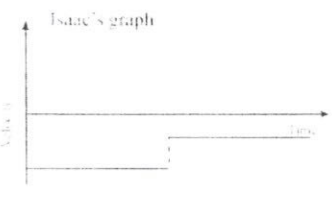
6. **Reason.** Is the change in velocity positive or negative? What does this tell us about the acceleration?

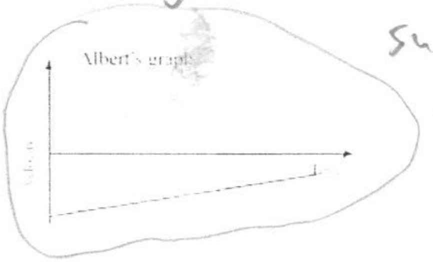
negative – tells us acceleration is negative.

7. **Reason.** Isaac and Albert draw a velocity graph to match the position graph on the left. Which velocity graph do you think best matches the position graph? Explain.

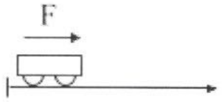
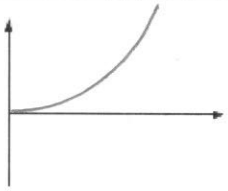
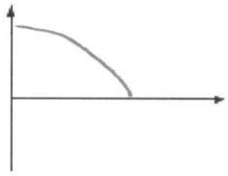
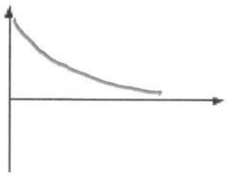
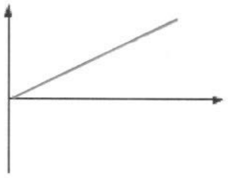
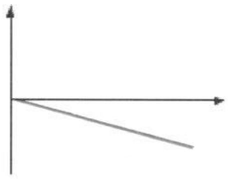
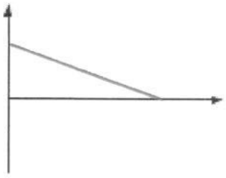
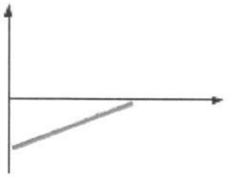
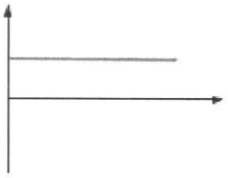
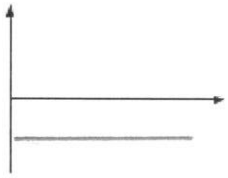
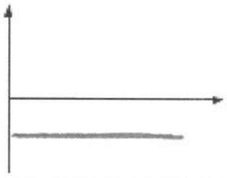
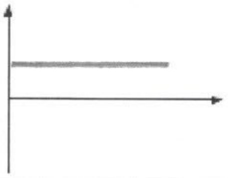
Velocity is changing gradually, not suddenly.







→ +

	1	2	3	4
Description	The cart is released from rest near the motion detector. The fan pushes on the cart **away** from the detector.	The cart is released from rest far from the detector. The fan pushes **towards** the detector.	The cart is moving away from the detector. The fan pushes **towards** the detector.	The cart is moving towards the detector. The fan is pushing **away** from the detector.
Sketch with Force				
Position graph				
Velocity graph				
Acceleration graph				
Slowing down or speeding up?	speeding up	speeding up	slowing down	slowing down
Sign of Velocity	+	−	+	−
Sign of Acceleration	+	−	−	+

Acceleration is a **vector** quantity, so the sign indicates a direction. This is **not** the direction of the object's motion!

2. **Reason.** Emmy says, "We can see from these results that when the acceleration is positive, the object always speeds up." Do you agree with Emmy? Explain. No. In case 4, the cart has a positive acceleration but is slowing down.

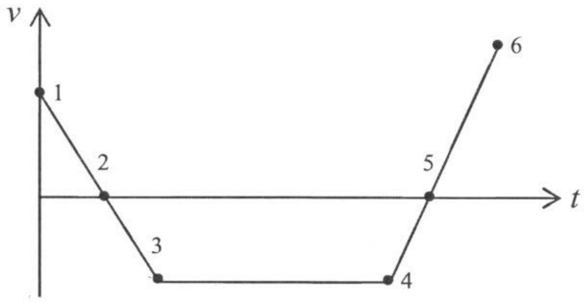
3. **Reason.** What conditions for the acceleration and velocity must be true for an object to be speeding up? To be slowing down?
Signs must be same to be speeding up.
" " " opposite " " slowing down.

4. **Reason.** Which quantity in our chart above does the sign of the acceleration **always** match?
Direction of fan: away → positive acceleration towards → negative acceleration.

Always compare the magnitudes of the velocities, the speeds, using the terms *faster* or *slower*. Describe the motion of accelerating objects as *speeding up* or *slowing down* and state whether it is moving in the positive of negative direction. **Never** use the d-word, *deceleration* - yikes! Note that we will always assume the acceleration is uniform (constant).

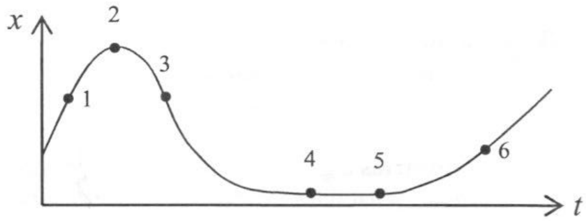
Homework: Speeding Up and Slowing Down

1. **Interpret and Explain.** A person walks back and forth in front of a motion detector producing the velocity graph shown to the right. Six events have been labelled on the graph. The chart below lists different examples of motion. Find the appropriate interval(s) of time in the graph that correspond to that type of motion and **provide evidence from the graph** supporting your choice.



Type of motion	Interval(s)	Evidence
positive acceleration	4-5 5-6	velocity is increasing
negative acceleration and a positive velocity	1-2	velocity is positive but decreasing ↳ negative acceleration
acceleration of zero	3-4	velocity is constant
speeding up	2-3 5-6	→ velocity is negative, as is acceleration → velocity is positive, as is acceleration
slowing down	1-2 4-5	→ velocity is positive but decreasing → velocity is negative but increasing
at rest (reminder: at rest means not moving for an **interval** of time)	None	no periods of time with $\vec{v} = 0$
Change of acceleration	Moments: 3, 4	3 → negative acceleration stops 4 → positive acceleration begins

2. **Interpret and Explain.** In a different experiment, a person walks back and forth in front of a motion detector and produces the position graph shown to the right. The chart below lists different examples of motion. Find the appropriate interval(s) of time or events in the graph that correspond to that type of motion and **provide evidence from the graph** supporting your choice.



Type of motion	Intervals or Events	Evidence
Zero velocity	4-5	position is not changing
Speeding up	2-3 5-6	→ getting steeper towards sensor → curve getting steeper (moving faster away)
Slowing down	1-2 3-4	→ moving away but levelling off → moving towards but levelling off
Turning around	2	changes from x increasing to decreasing

4-5 → long pause but resumes in opposite direction.

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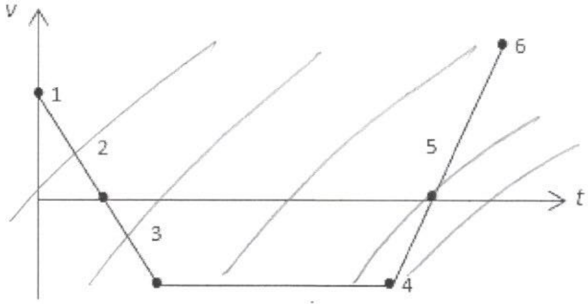
SPH3U Area and Displacement



A graph is more than just a line or a curve. We will discover a very handy new property of graphs which has been right under our noses (and graphs) all this time!

A: Looking Under the Graph

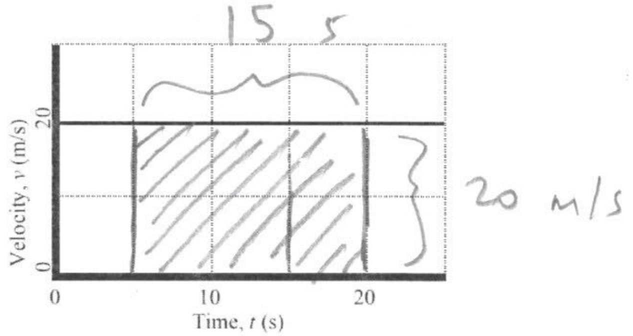
A car drives south along a straight road at 20 m/s. After 5 s the car passes a streetlight and at 20 s the car passes a bus stop.

1. **Calculate** the displacement of the car between the streetlight and the bus stop using the formula $\vec{v} = \Delta\vec{x}/\Delta t$

$\vec{v} = \frac{\Delta x}{\Delta t}$ $\quad 20 = \frac{\Delta x}{15}$ $\quad \boxed{\Delta x = 300\ m}$

2. **Sketch.** Now we will think about this calculation in a new way. Draw and shade a rectangle on the graph that fills in the area between the line of the graph and the time axis, for the time interval of 5 to 20 seconds.

3. **Interpret.** Calculate the area of the rectangle. Note that the length and width have a meaning in physics, so the final result is not a physical area. Use the proper physics units that correspond to the height and the width of the rectangle. What physics quantity does the final result represent?

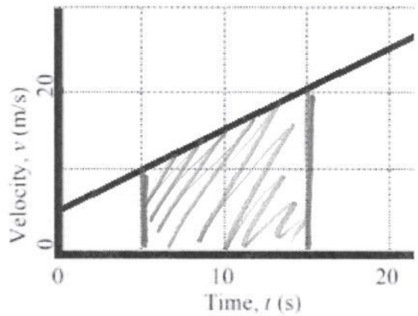


$(20\ \frac{m}{s})(15\ s) = 300\ m$

Area under a velocity graph. The *area* under a velocity-time graph for an interval of motion gives the *displacement* during that interval. Both velocity and displacement are vector quantities and can be positive or negative depending on their directions. According to our usual sign convention, areas above the time axis are positive and areas below the time axis are negative.

B: What if Velocity is not Constant?

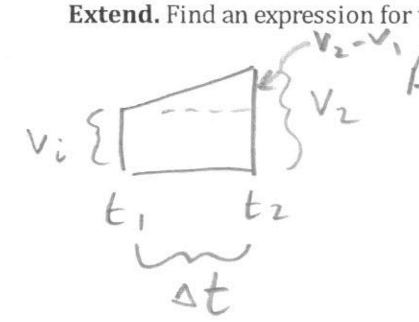
Consider the velocity time graph shown in the diagram.
Suppose we want to know how far the car travelled between $t_1 = 5.0$ s and $t_2 = 15.0$ s. Shade in the area representing the distance travelled over this interval.



Calculate the area of your shaded region. Remember to include units.

A = ▭ + ◺
= (10)(10) + ½(10)(10)
= 100 + 50
A = 150 m/s

Extend. Find an expression for the area under the graph using some or all of the variables t_1, t_2, Δt, $\vec{v}_1$, $\vec{v}_2$



A = ▭ + ◺

$= (\Delta t)(\vec{v}_1) + \frac{1}{2}\Delta t(\vec{v}_2 - \vec{v}_1)$

$= \vec{v}_1 \Delta t + \frac{1}{2}\vec{v}_2 \Delta t - \frac{1}{2}\vec{v}_1 \Delta t$

$= \frac{1}{2}\vec{v}_1 \Delta t + \frac{1}{2}\vec{v}_2 \Delta t$ $\rightarrow = \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)\Delta t$