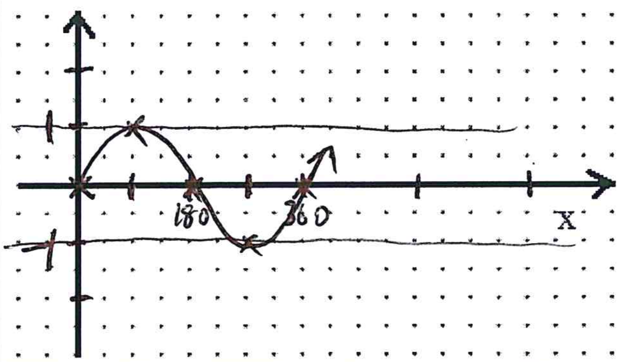
-1-

SOLUTIONS

Applying Transformations to Sinusoidal Functions

Carefully graph $f(x) = \sin x$ and $f(x) = \cos x$. Identify key features and state the key points.

f(x) = sin(x)



"Key" Key features (axis, amplitude, period, phase shift, reflection)

IDENTIFY!

Axis: Y= 0

Amplitude: 1

Period: 360

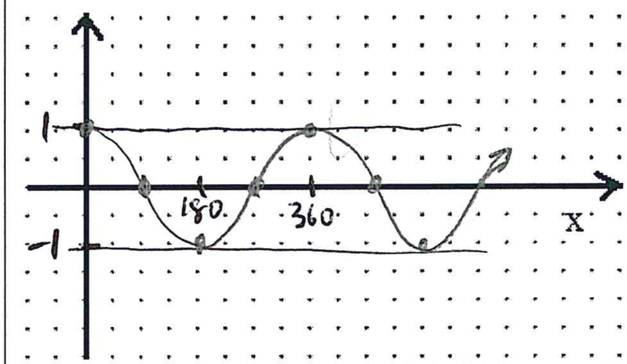
Phase shift: None
↳ "TRANSLATION" Left/Right.

Reflection over axis? no.

Key Points

x	f(x)
0	0
90	1
180	0
270	-1
360	0

f(x) = cos(x)



GRAPH ME!

"Key" Key features (axis, amplitude, period, phase shift, reflection)

Axis: Y= 0

Amplitude: 1

Period: 360

Phase shift: None
↳ TRANSLATION LEFT/RIGHT

Reflection over axis? no.

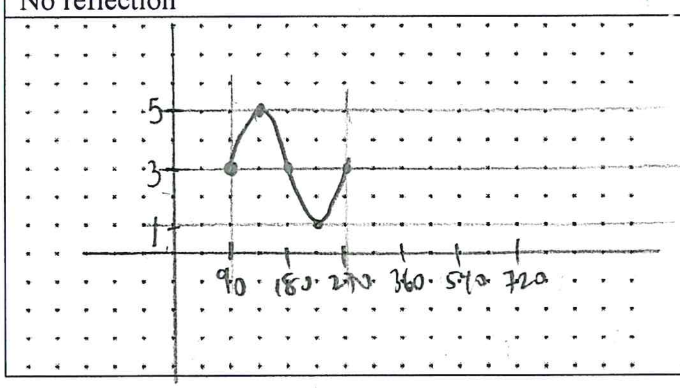
Key Points

x	f(x)
0	1
90	0
180	-1
270	0
360	1

Now carefully draw the following.

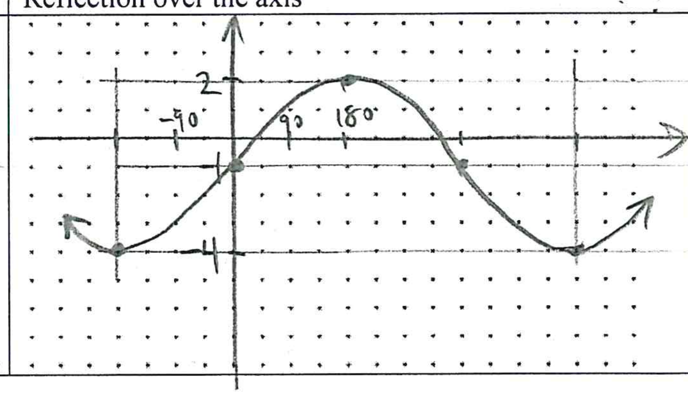
A function based on $f(x) = \sin x$, but with:

Axis: 3
Amplitude: 2
Period: 180
Phase shift: 90 degrees to the right
No reflection



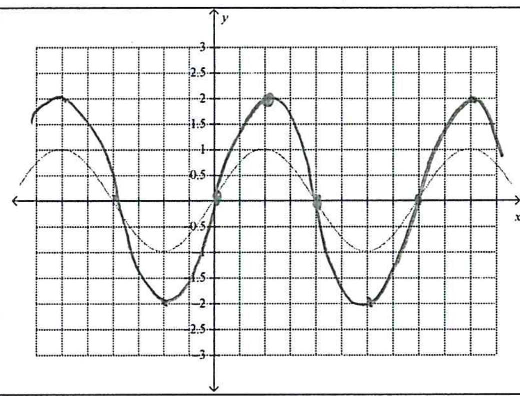
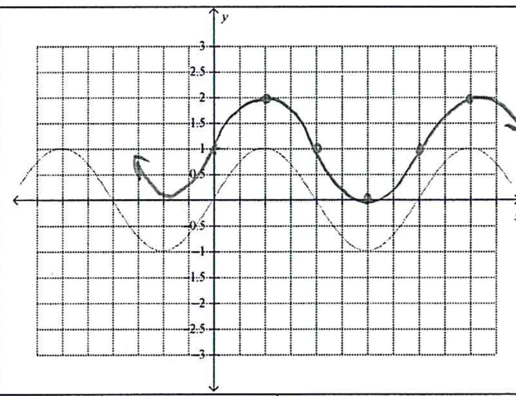
A function based on $f(x) = \cos x$, but with:

Axis: -1
Amplitude: 3
Period: 720
Phase shift: 180 degrees to the left
Reflection over the axis



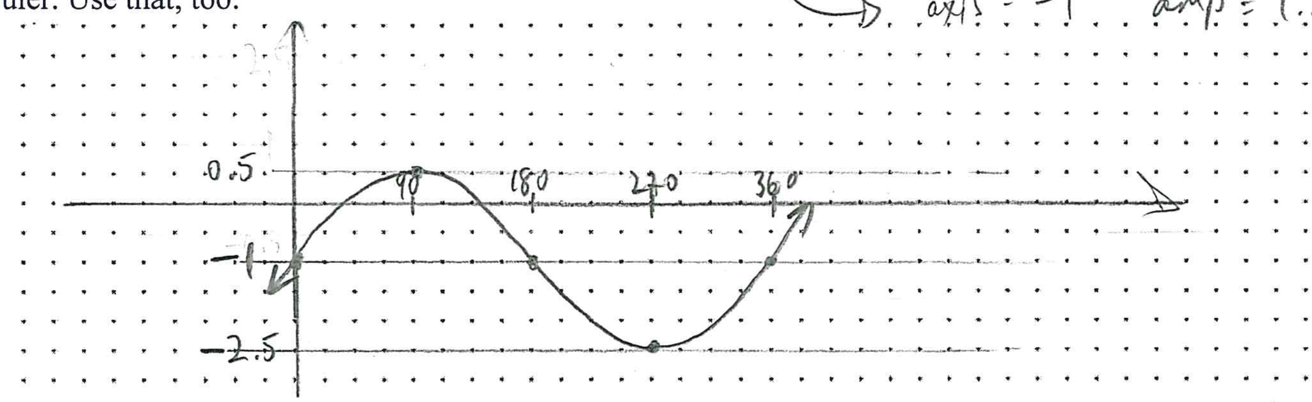
Now we will transform $f(x) = \sin x$ and $f(x) = \cos x$. The goal is to be able to sketch something like $g(x) = -2\sin(0.5x - 90) + 1$ without too much difficulty.

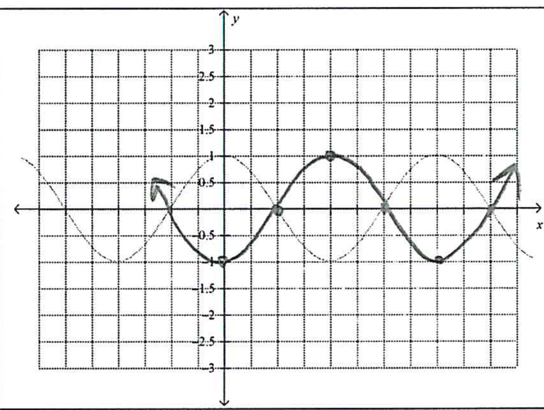
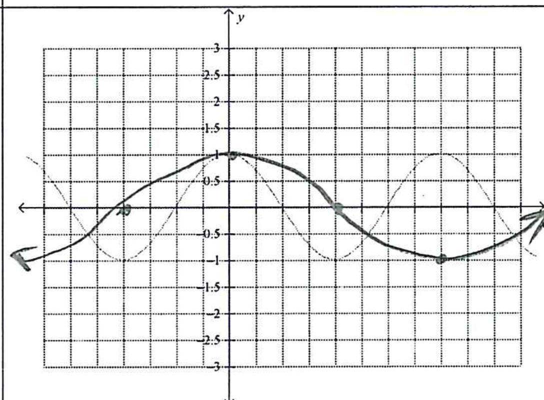
<u>Pay attention to connections between the equations and the key features.</u>

Parent Function	$f(x)=\sin x$	$f(x)=\sin x$
Transformed function $g(x)$	$g(x)=2\sin x$	$g(x)=\sin x+1$
$g(x)$ in terms of $f(x)$	$g(x)=2f(x)$	$g(x)=f(x)+1$
Input Output Diagram	$x \rightarrow \boxed{f} \rightarrow \boxed{\times 2} \rightarrow g$ 0 0 0 90 1 2 180 0 0 270 -1 -2 360 0 0	$x \rightarrow \boxed{f} \rightarrow \boxed{+1} \rightarrow g$
Description of transformations (in the correct order)	v. stretch by 2	translate up 1
Graph		
Key Features	Axis: y=0 Amplitude: 2 Period: 360 Phase shift: none Reflection over axis? no	Axis: y=1 Amplitude: 1 Period: 360 Phase shift: none Reflection over axis? no

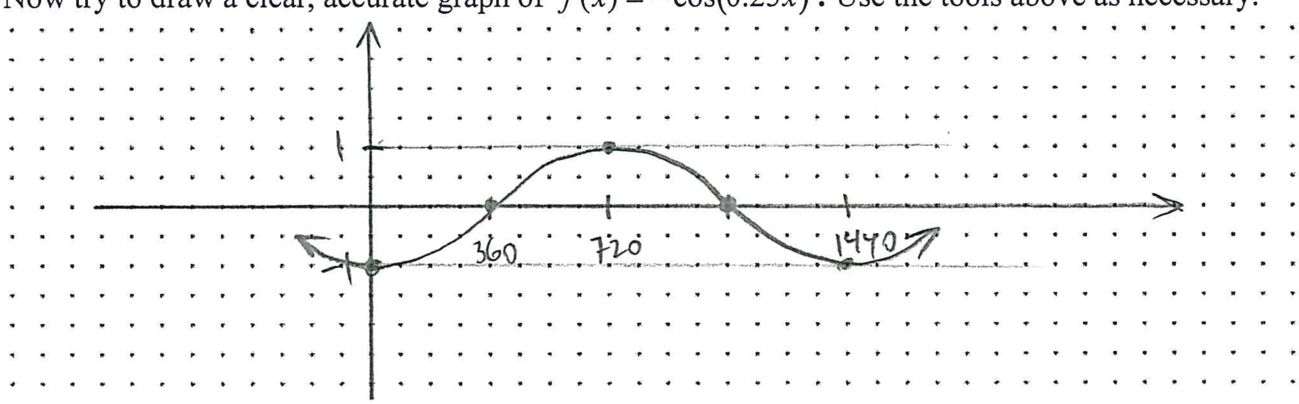
Now try to draw a clear, accurate graph of $f(x)=1.5\sin x-1$. Use the tools above as necessary. And a ruler. Use that, too.

→ axis = −1 amp = 1.5



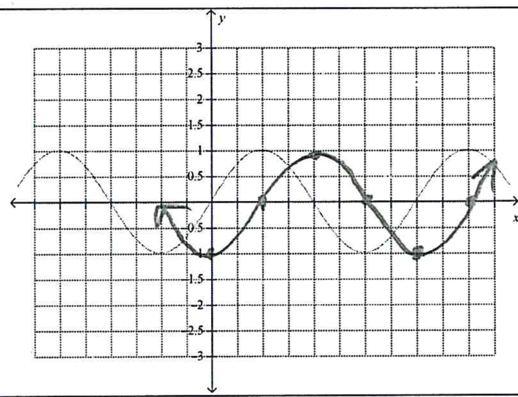
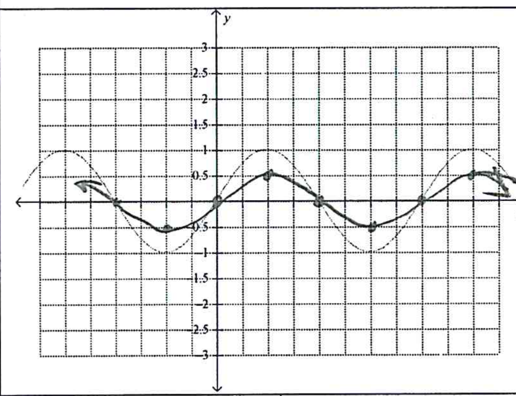
Parent Function	$f(x) = \cos x$ ← new parent function!	$f(x) = \cos x$
Transformed function g(x)	$g(x) = -\cos x$	$g(x) = \cos(0.5x)$
g(x) in terms of f(x)	$g(x) = -f(x)$	$g(x) = f(0.5x)$
Input Output Diagram	$x \to \boxed{f} \to \boxed{\times(-1)} \to g$	$x \to \boxed{\times 0.5} \to \boxed{f} \to g$ 0 0 1 180 90 0 360 180 -1 540 270 0 720 360 1
Description of transformations (in the correct order)	reflect over x	h. stretch by 2
Graph		
Key Features	Axis: y=0 Amplitude: 1 Period: 360 Phase shift: no Reflection over axis? yes	Axis: y=0 Amplitude: 1 Period: 720 Phase shift: none Reflection over axis? no

Now try to draw a clear, accurate graph of $f(x) = -\cos(0.25x)$. Use the tools above as necessary.



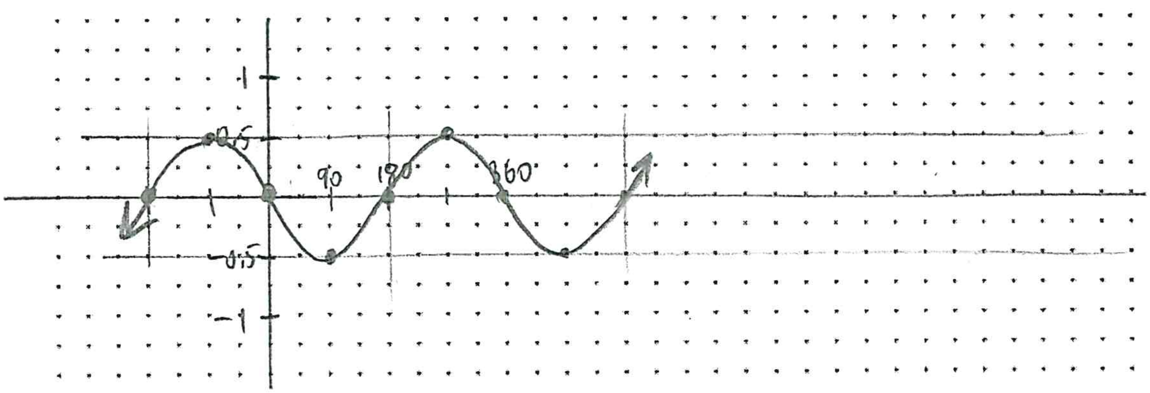
$f(x) = -\cos(0.25x)$

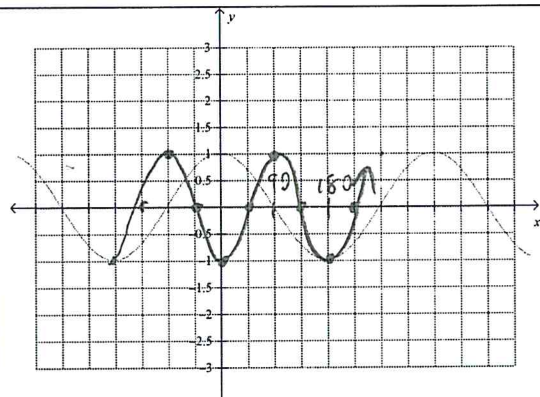
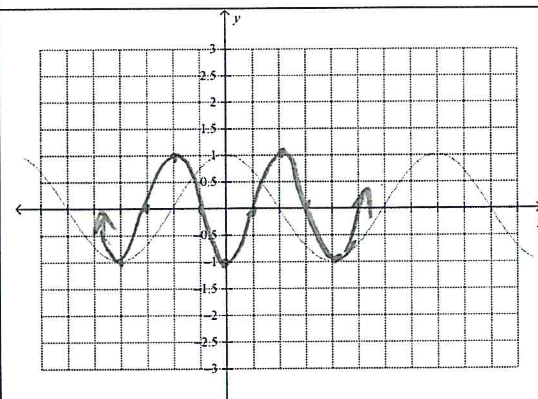
- reflect   - period $= \frac{360}{0.25} = 1440$

Parent Function	$f(x)=\sin x$	$f(x)=\sin x$
Transformed function $g(x)$	$g(x)=\sin(x-90)$	$g(x) = \frac{1}{2}\sin x$
$g(x)$ in terms of $f(x)$	$g(x) = f(x-90)$	$g(x)=\frac{1}{2}f(x)$
Input Output Diagram	$x \to \boxed{-90} \to \boxed{f} \to g$	$x \to \boxed{f} \to \boxed{\times\frac{1}{2}} \to g$
Description of transformations (in the correct order)	- phase shift right 90°	v. compress by 2
Graph		
Key Features	Axis: $y=0$ Amplitude: 1 Period: 360 Phase shift: 90 right Reflection over axis? no	Axis: $y=0$ Amplitude: 0.5 Period: 360 Phase shift: none Reflection over axis? no

Now try to draw a clear, accurate graph of $f(x)=0.5\sin(x+180)$. Use the tools above as necessary.

amp = 0.5
shift: left 180
period = 360



Parent Function	$f(x)=\cos x$	$f(x)=\cos x$
Transformed function $g(x)$	$g(x)=\cos(2x+180)$	$g(x)=\cos[2(x+90)]$
$g(x)$ in terms of $f(x)$	$g(x)=f(2x+180)$	$g(x)=f[2(x+90)]$
Input Output Diagram	$x \to \boxed{\times 2} \to \boxed{+180} \to \boxed{F} \to g$ -90 -180 0 1 -45 -90 90 0 0 0 180 -1 45 90 270 0 90 180 360 1	$x \to \boxed{+90} \to \boxed{\times 2} \to \boxed{F} \to g$
Description of transformations (in the correct order)	- left 180 - h. compress by 2	- h. compress by 2 - left 90
Graph		
Key Features	Axis: $y=0$ Amplitude: 1 Period: $\frac{360}{2}=180$ Phase shift: left 90 Reflection over axis? no	Axis: $y=0$ Amplitude: 1 Period: 180 Phase shift: left 90 Reflection over axis? no

ALRIGHT STOP!! THE TWO GRAPHS ABOVE SHOULD BE IDENTICAL. IF THEY ARE NOT, THERE'S A MISTAKE. NOTE THE TWO FORMATS:

$$g(x)=\cos(2x+180) \quad vs \quad g(x)=\cos[2(x+90)]$$

BOTH CAN BE USED TO SKETCH A GRAPH, BUT ONE FORMAT IS MOST USEFUL IN IDENTIFYING THE PHASE SHIFT OF THE GRAPH (ie. 90° LEFT).

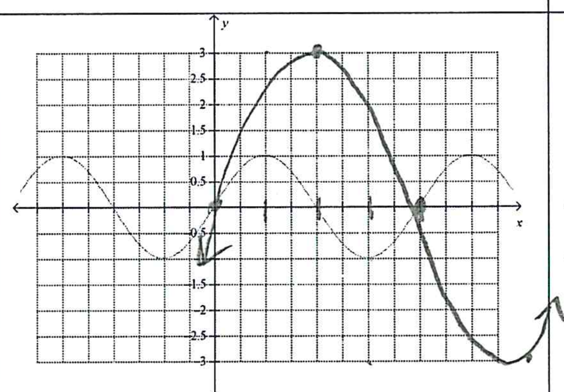
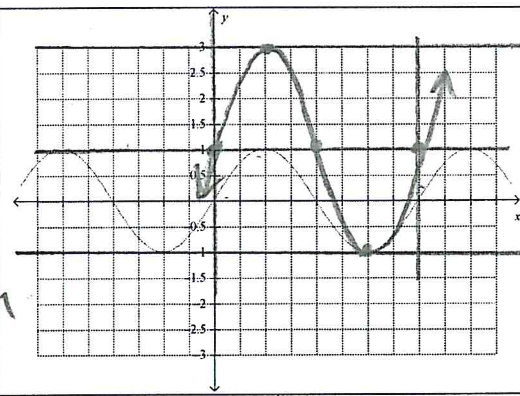
Change the following into a form that is more useful in identifying the phase shift.

$g(x)=\sin(2x-360)$

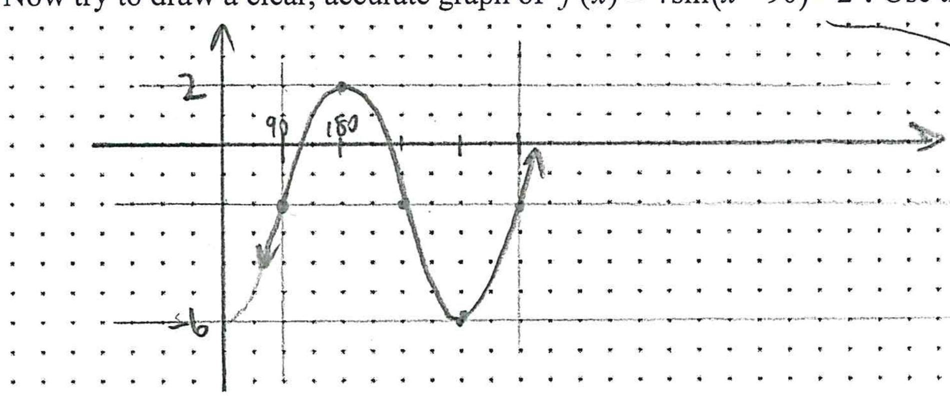
$=\sin[2(x-180)]$

$g(x)=\cos(0.5x+90)$

$=\cos[0.5(x+180)]$

Parent Function	$f(x) = \sin x$	$f(x) = \sin x$
Transformed function *g(x)*	$g(x) = 3\sin(0.5x)$	$g(x) = -2\sin x + 1$
g(x) in terms of *f(x)*	$g(x) = 3f(0.5x)$	$g(x) = -2f(x) + 1$
Input Output Diagram	$x \rightarrow \boxed{\times 0.5} \rightarrow \boxed{f} \rightarrow \boxed{\times 3} \rightarrow g$	$x \rightarrow \boxed{f} \rightarrow \boxed{\times 2} \rightarrow \boxed{\times(-1)} \rightarrow \boxed{+1} \rightarrow g$
Description of transformations (in the correct order)	v. stretch by 3 h. stretch by 2 could reverse	-v. stretch by 2 -reflect over x -up 1
Graph		
Key Features	Axis: $y=0$ Amplitude: 3 Period: $\frac{360}{0.5} = 720$ Phase shift: none Reflection over axis? no	Axis: $y=1$ Amplitude: 2 Period: 360 Phase shift: none Reflection over axis? yes.

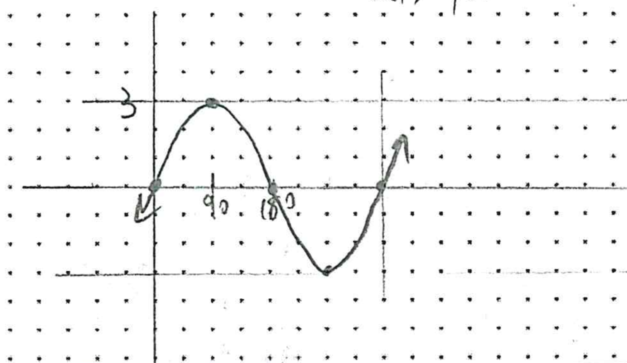
Now try to draw a clear, accurate graph of $f(x) = 4\sin(x - 90) - 2$. Use the tools above as necessary.



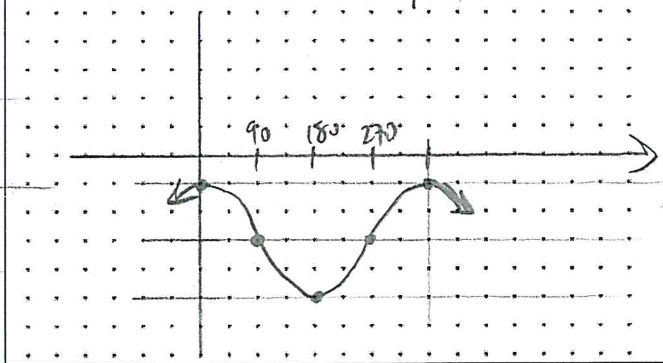
amp = 4
shift right 90
down 2
period = 360

Draw a clear, accurate graph of each of the following:

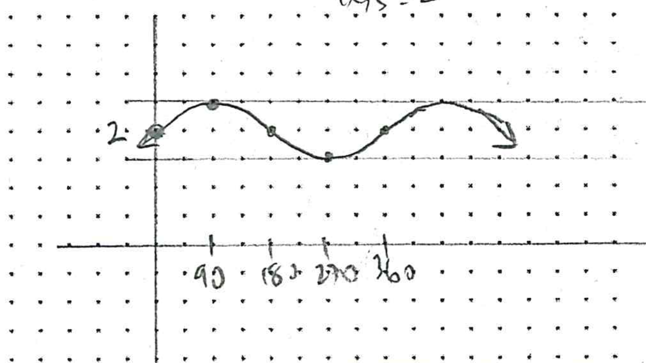
a) $f(x) = 3\sin x$ amp = 3 per. = 360 axis y = 0



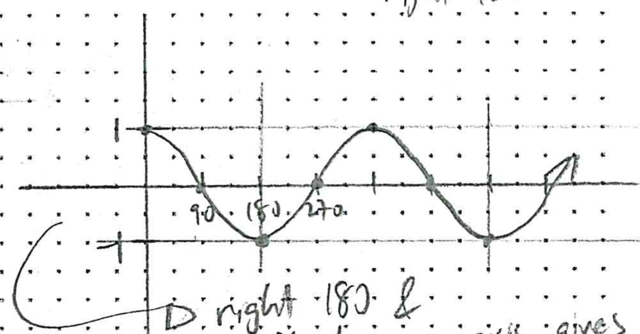
b) $f(x) = \cos x - 1.5$ axis y = −1.5 amp = 1 per. = 360



c) $f(x) = 0.5\sin x + 2$ amp = 0.5 axis = 2 per = 360

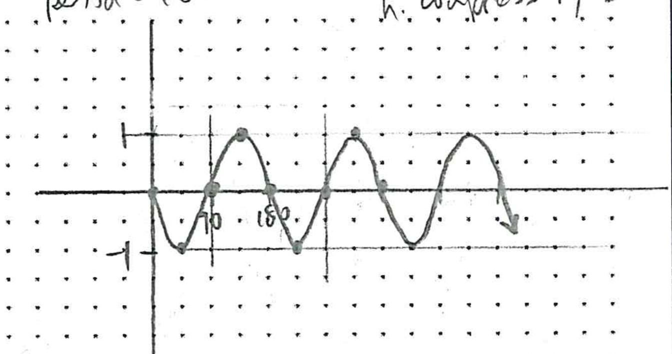


d) $f(x) = -\cos(x - 180)$ reflect over axis right 180 amp = 1 axis y = 0

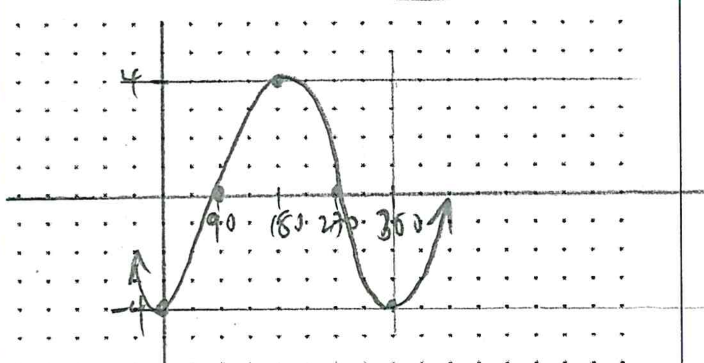


D. right 180 & reflect over axis gives original cosine curve.

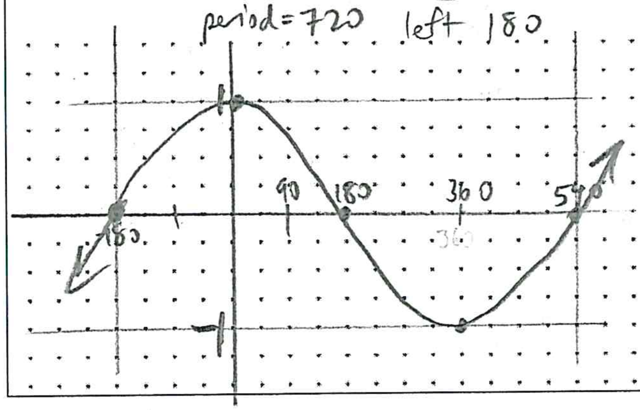
e) $f(x) = \sin[2(x - 90)]$ right 90 period = 180 h. compress by 2



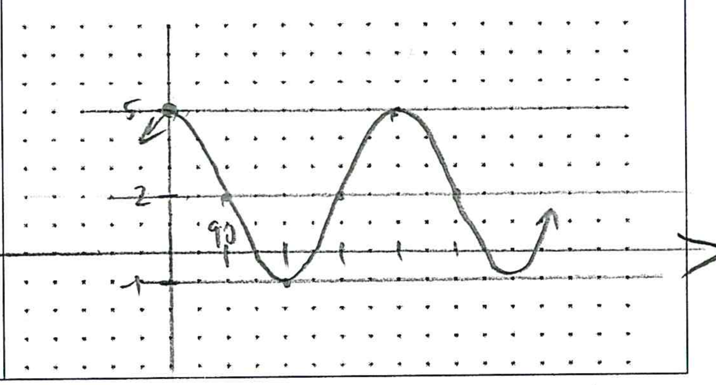
f) $f(x) = -4\cos x$



g) $f(x) = \sin(0.5x + 90)$ = sin[0.5(x + 180)] period = 720 left 180



h) $f(x) = 3\cos x + 2$

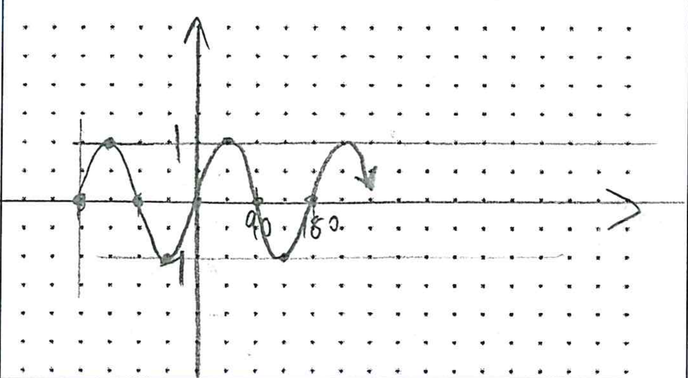


period = 180   left 180
= sin[2(x+180)]

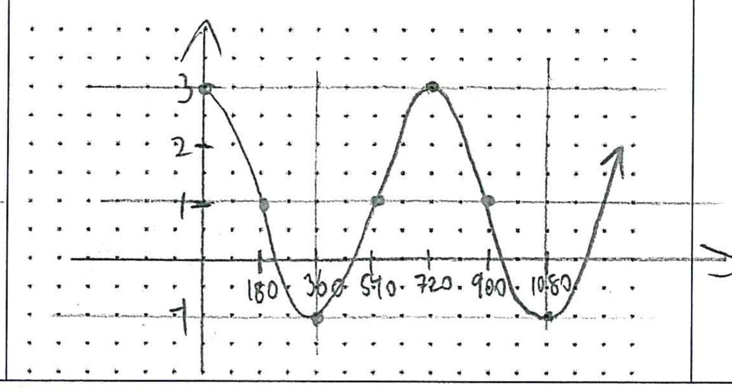
axis = 1   period = 720   right 360, reflect over axis
= -2cos[0.5(x-360)] + 1

amp = 2

i) $f(x) = \sin(2x + 360)$

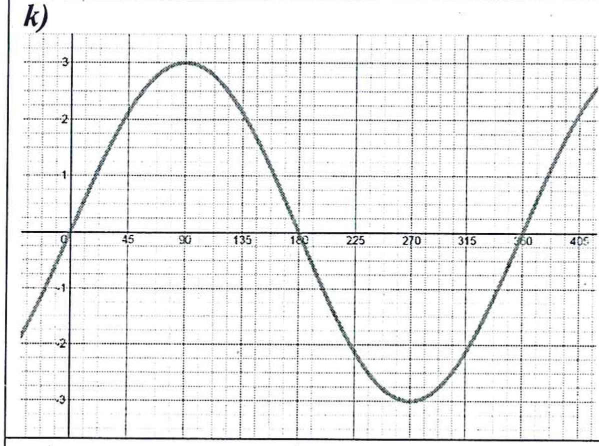


j) $f(x) = -2\cos(0.5x - 180) + 1$

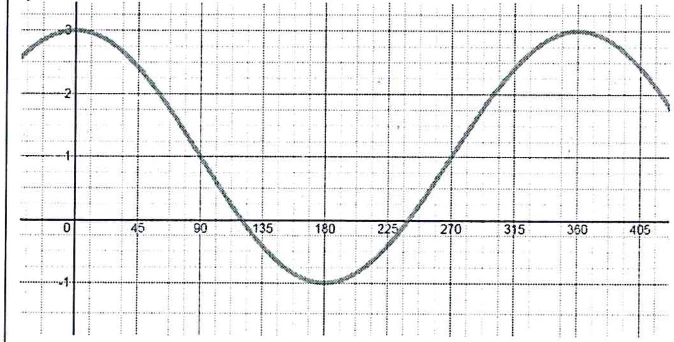


Now determine two equations for each of the following curves: one using $\sin x$ ***as a parent function, the other using*** $\cos x$

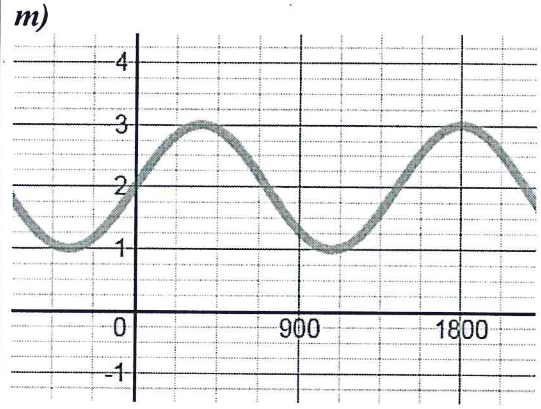
k)



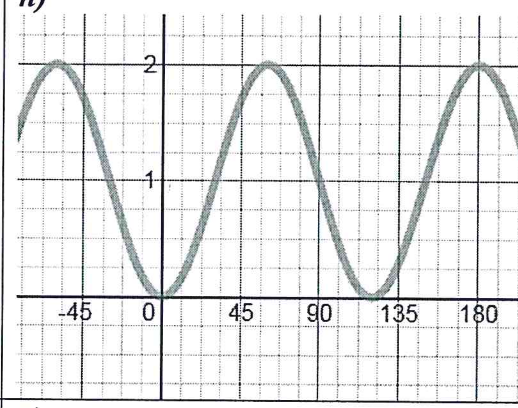
l)



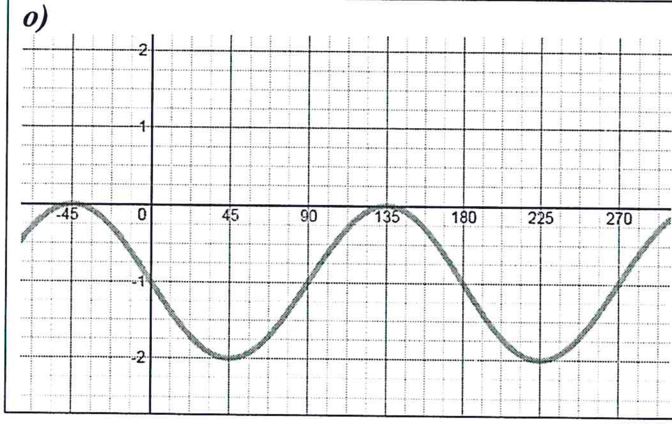
m)



n)



o)



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