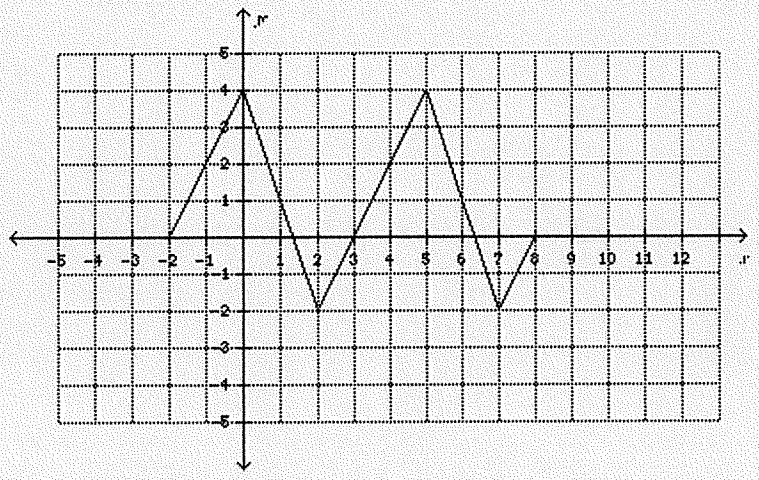
MCR3U: Periodic Functions – A Graphical Approach (ANSWERS)

Key features of a periodic function:
-y-intercept -axis -period
-amplitude -domain/range

A ***periodic*** function is one that repeats its behaviour identically in cycles.



1. a) How many ***cycles*** are shown in the graph?
There are 2 cycles

b) What is the ***period*** (length of one cycle)?
Period = 5

c) What are the ***maximum*** and ***minimum*** values?
Maximum is y = 4, Minimum is y = -2

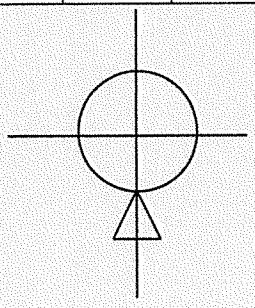
d) What is the ***amplitude***?
Amplitude is 3

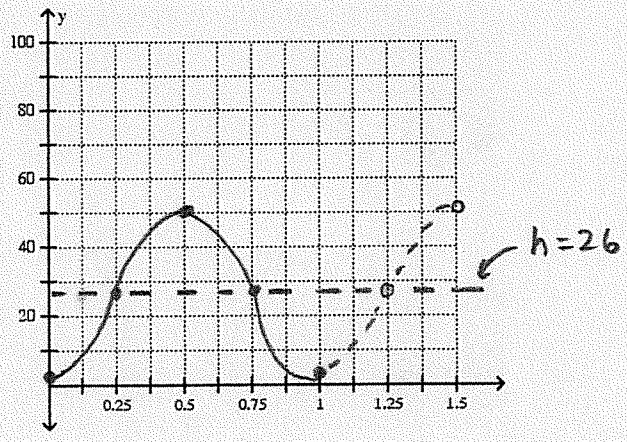
e) What is the ***equation of the axis***?
Equation of the axis is y = 1

f) Suppose the periodic behaviour continues. Then, what is the value of:
i) $f(12) = -2$ **ii)** $f(-4) = 1$ **iii)** $f(35) = 4$

2. A Ferris wheel has a diameter of 50 m, and stands 1 m above the ground. As it rotates, the height of a particular seat above the ground is changing. It takes 1 minute for the Ferris wheel to make 1 complete revolution. **Complete** the table and **sketch** a graph.

Time (min)	0	0.25	0.5	0.75	1.0
Height (m)	**1**	**26**	**51**	**26**	**1**





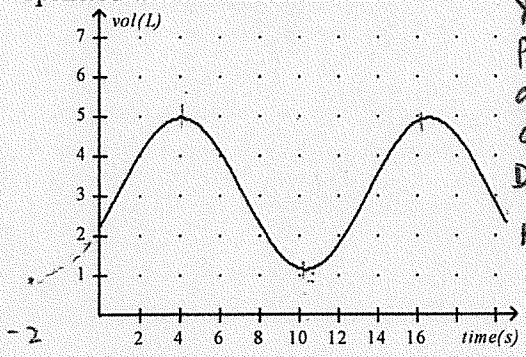
In the above graph, what is the ***equation of the axis***?

h = 26

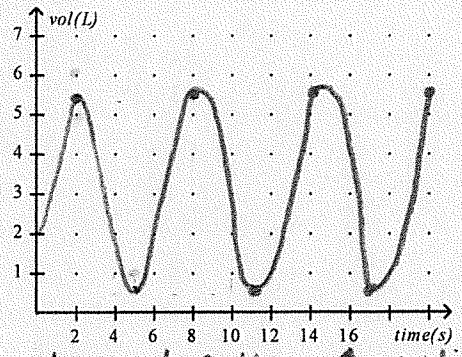
3. The graph below shows the volume of air in a person's lungs at rest.

a) State the key features of the graph, and interpret their meaning in the context of the question.

b) Sketch a new graph which shows what this person's breathing curve might look like after they just sprinted across a soccer field.



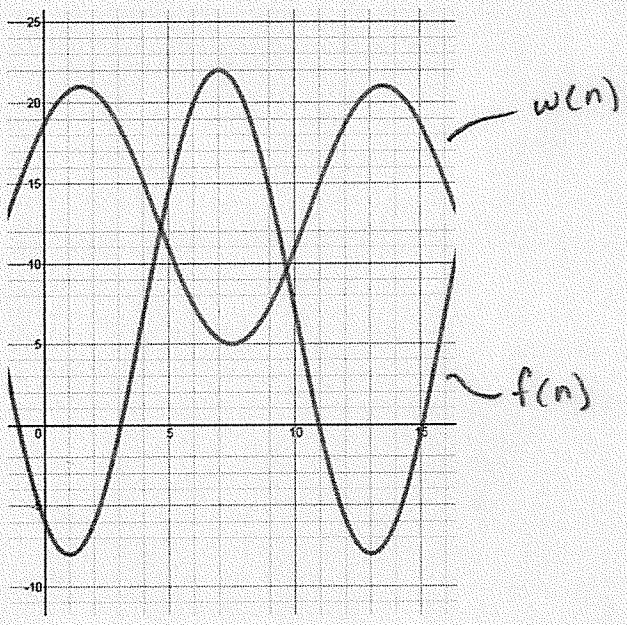
y-int = 2.5
Period = 12
axis: y = 3
amplitude = 2
$D = \{t \in R \mid t \geq 0\}$
$R = \{v \in R \mid 1 \leq v \leq 5\}$



- deeper breaths – ↑ amplitude.
- quicker breaths – shorter period

4. Use Desmos to help you answer the following:

The function $f(n) = 15\cos(30n - 210) + 7$ gives the relationship between the average temperature in Ottawa in degrees Celsius, and the month number of the year, n (January = 0, February = 1, etc). The function $w(n) = 8\sin(30n + 45) + 13$ gives similar information for Wellington, New Zealand.



a) Determine the warmest and coldest times of year in Wellington.

Warmest is winter, coldest is summer

b) Determine the **period** of each function. **Explain** why this period makes sense in this context.

The period of both functions is 12 months … this makes sense because after 12 months a new year will start and weather tends to be similar each month over the years

c) Determine the temperature difference between the warmest and coldest times of year in Ottawa.

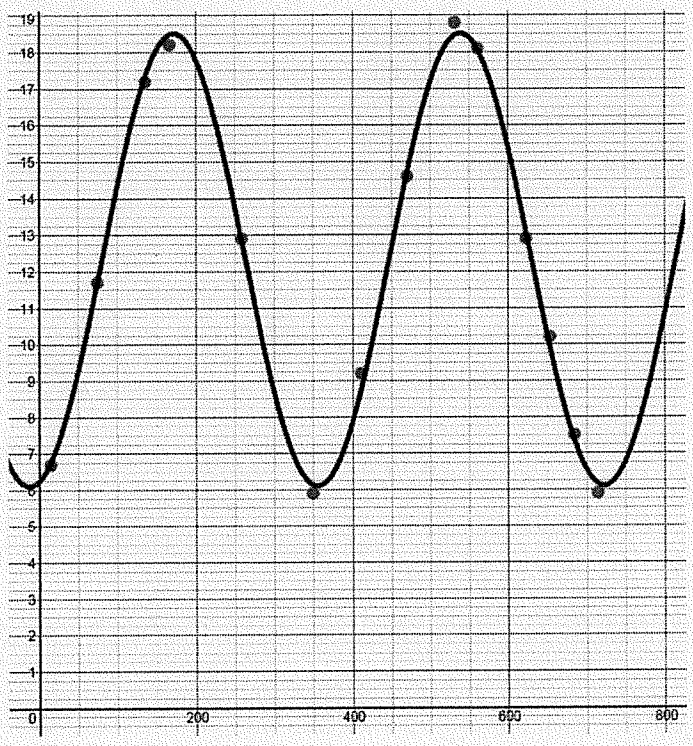
30 degrees Celsius

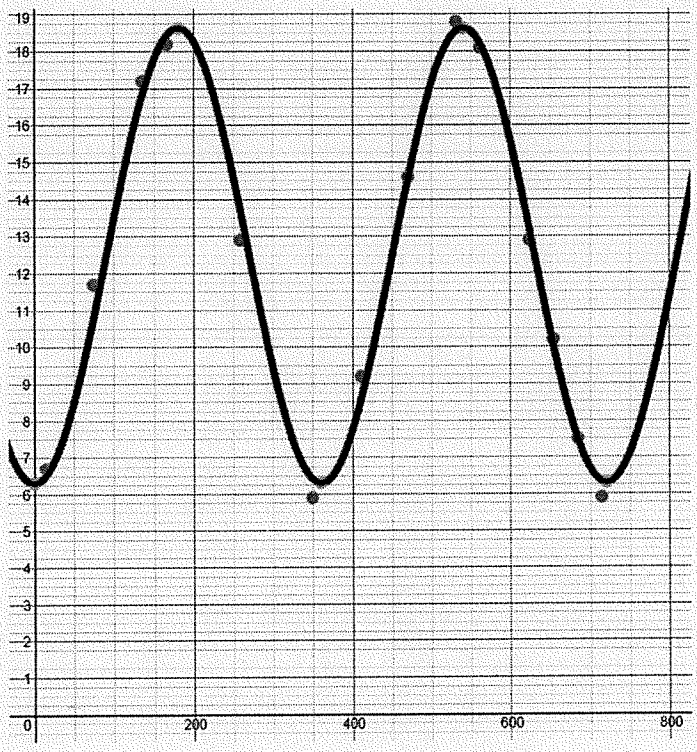
d) Over which months in a given year is it warmer in Ottawa than in Wellington? **May to October**

5. For a particular location, the number of hours of sunlight is shown for certain days over a two-year period starting January 1st, 2010. Day 366 is January 1st, 2011.

Day #	15	74	135	166	258	349	411	470	531	561	623	653	684	714
Hours of sunlight	6.7	11.7	17.2	18.2	12.9	5.9	9.2	14.6	18.8	18.1	12.9	10.2	7.5	5.9

$y \sim a\sin(x - d) + q \rightarrow y = -6.14\sin(x - 266) + 12.5$ **OR** **$y \sim a\cos(x) + q \rightarrow y = -6.16\cos(x) + 12.5$**





sin(x) and cos(x) are 2 forms of periodic functions that we will study … they are very similar … can you figure out how they are different by looking at the equations?

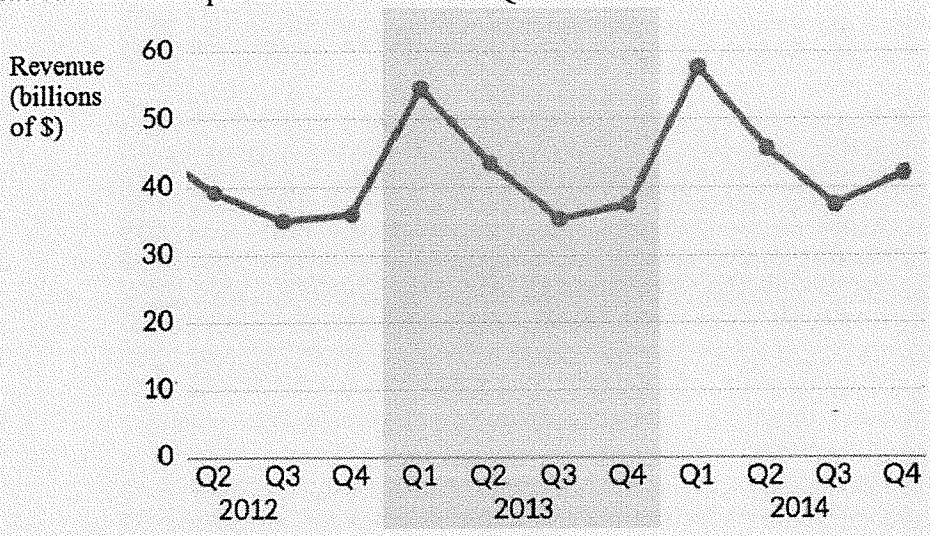
How many hours of sunlight do you expect on August 3rd, 2012? (**hint**…what day # of the year is this?)

August 3 is about the 272nd day of the year, so x = 272 … now trace it on the graph to find y-value
Answer: about 12.2 hours

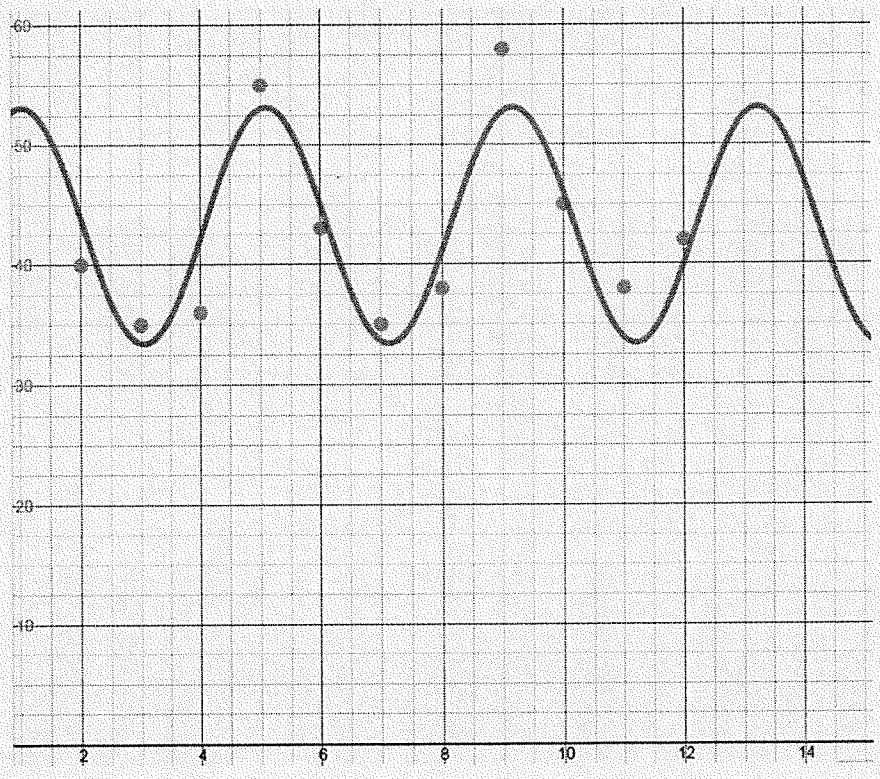
On which date in 2012 would you expect 8 hours of sunlight?

43rd (February 12) and 316th day (November 13) of the year

6. The graph below shows the revenue, in billions of $, of Apple over a period of time. Use a mathematical model to predict its revenue for Q1 of 2015.



(Apple's actual revenue in Q1 of 2015 was $74.6 billion. Shows how useful mathematical models are!)



let x represent the successive quarters from 2012 to 2014
let y represent revenue in billions of $

Model: $y = -9.87\sin(-1.54x) + 43.3$

Using the model, the prediction for Q1 in 2015 (x = 13) is around $52.5 billion.