R 3U Transformations of Exponential Functions

SOLUTIONS

Exponential functions have many different parent functions. $f(x)=2^x$, $f(x)=3^x$, $f(x)=\left(\frac{1}{2}\right)^x$, etc. are all different parent functions with their own set of key points. You need to be able to generate these quickly.

Parent function: $f(x)=2^x$

Key Points:

x	f(x)
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Parent function: $f(x)=3^x$

Key Points:

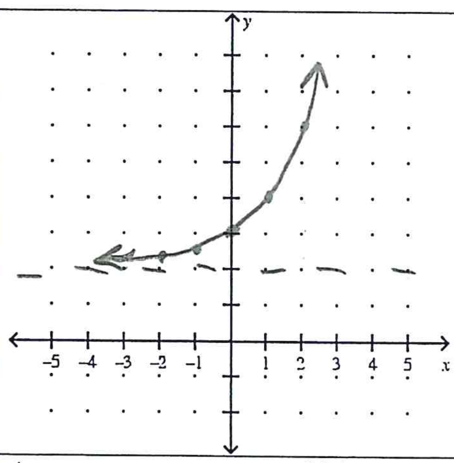
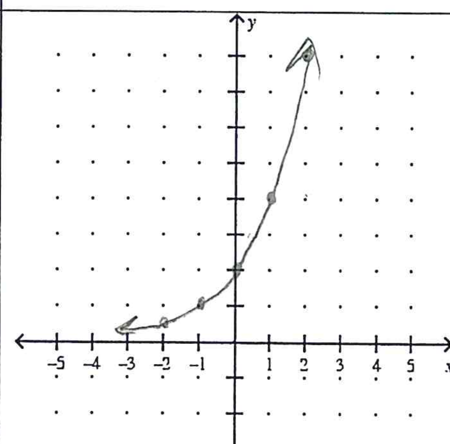
x	f(x)
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

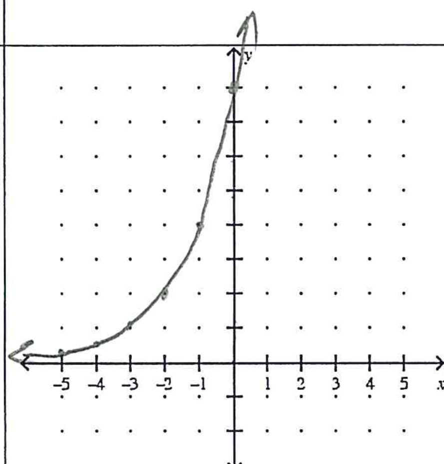
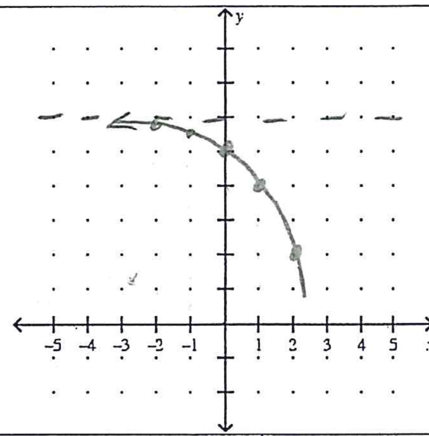
Parent function: $f(x)=\left(\frac{1}{2}\right)^x$

Key Points:

x	f(x)
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

Guess what…the way we apply transformations to exponential functions is the same as how we apply transformations to quadratic, square root, sinusoidal, and all other functions. That's good news. Right?

Parent Function	$f(x)=2^x$	$f(x)=2^x$
Transformed function $g(x)$	$g(x)=2^x+2$	$g(x)=2(2)^x$
$g(x)$ in terms of $f(x)$	$g(x)=f(x)+2$	$g(x)=2f(x)$
Input Output Diagram	$x \to \boxed{f} \to \boxed{+2} \to g$ -2, $\frac{1}{4}$, 2.25 -1, $\frac{1}{2}$, 2.5 0, 1, 3 1, 2, 4 2, 4, 6	$x \to \boxed{f} \to \boxed{\times 2} \to g$
Description of transformations (in the correct order)	translate up 2	v. stretch by 2
Graph		
Key Features	Asymptote: $y=2$ Y-intercept: 3 Increasing/Decreasing: inc. Growth/Decay: growing	Asymptote: $y=0$ Y-intercept: 2 Increasing/Decreasing: inc. Growth/Decay: Grow

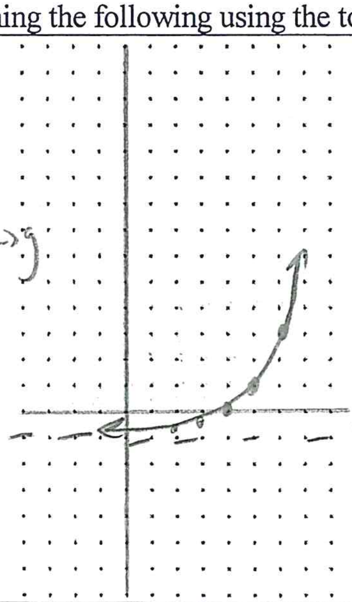
Parent	$f(x)=2^x$	$f(x)=2^x$
Transformed function $g(x)$	$g(x)=2^{x+3}$	$g(x) = -(2^x)+6$
$g(x)$ in terms of $f(x)$	$g(x) = f(x+3)$	$g(x)=-f(x)+6$
Input Output Diagram	$x \to \boxed{+3} \to \boxed{f} \to g$ -5, -2, 0.25 -4, -1, 0.5 -3, 0, 1 -2, 1, 2 -1, 2, 4	$x \to \boxed{f} \to \boxed{x(-1)} \to \boxed{+6} \to g$
Description of transformations (in the correct order)	left 3	-reflect over x -up 6
Graph		
Key Features	Asymptote: $2^3=8$ Y-intercept: $2^3=8$ Increasing/ Decreasing: inc Growth/Decay: grow	Asymptote: $y=6$ Y-intercept: 5 Increasing/ Decreasing: dec. Growth/Decay: grow

Now try carefully graphing the following using the tools and insights from above.

$g(x)=2^{x-4}-1$

$x \to \boxed{-4} \to \boxed{f} \to \boxed{-1} \to g$

- right 4
- down 1

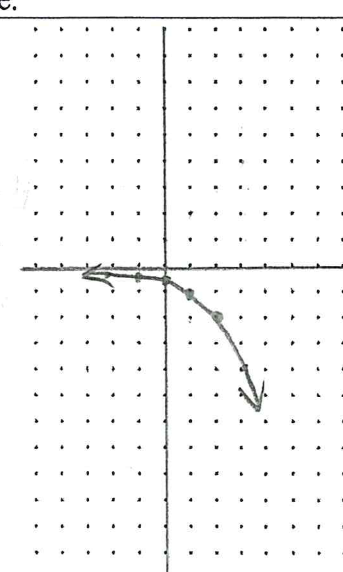


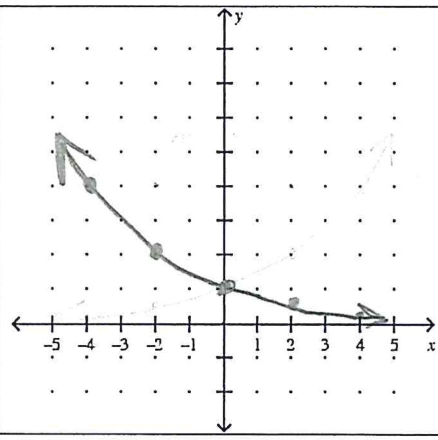
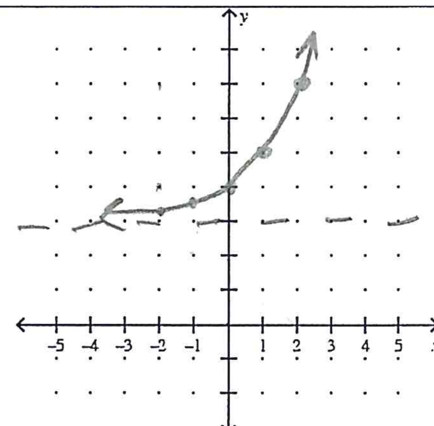
$y=-1$

$g(x)=-0.5(2)^x$

$x \to \boxed{f} \to \boxed{\times 0.5} \to \boxed{\times(-1)} \to g$

- v. compress by 2
- reflect over x



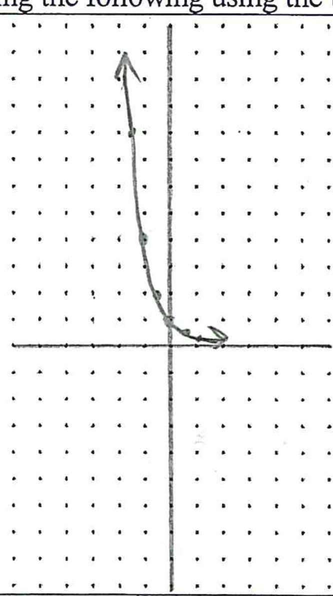
Parent	$f(x)=\left(\frac{1}{2}\right)^x$ ← CAREFUL! new parent function	$f(x)=\left(\frac{1}{2}\right)^x$
Transformed function $g(x)$	$g(x)=\left(\frac{1}{2}\right)^{0.5x}$	$g(x)=\left(\frac{1}{2}\right)^{-x}+3$
$g(x)$ in terms of $f(x)$	$g(x)=f(0.5x)$	$g(x)=f(-x)+3$
Input Output Diagram	$x \to \boxed{\times 0.5} \to \boxed{f} \to g$	$x \to \boxed{\times(-1)} \to \boxed{f} \to \boxed{+3} \to g$
Description of transformations (in the correct order)	h. stretch by 2	- up 3 - reflect over y
Graph		
Key Features	Asymptote: $y=0$ Y-intercept: 1 Increasing/ Decreasing: dec Growth/Decay: dec	Asymptote: $y=3$ Y-intercept: 4 Increasing/ Decreasing: inc Growth/Decay: grew

Now try carefully graphing the following using the tools and insights from above.

$$g(x)=\left(\frac{1}{2}\right)^{2x}$$

$x \to \boxed{\times 2} \to \boxed{f} \to g$

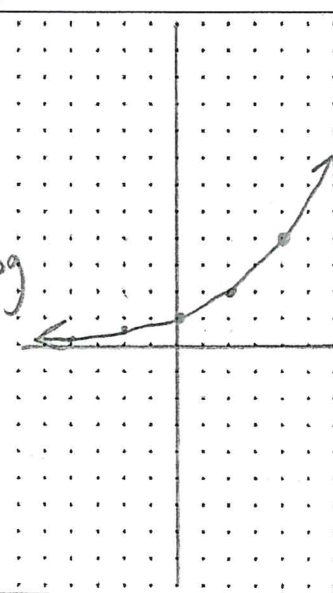
h. compress by 2

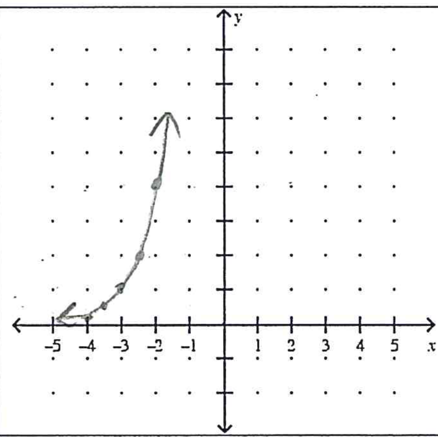
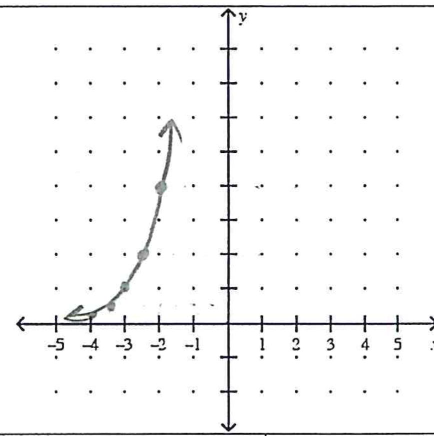


$$g(x)=\left(\frac{1}{2}\right)^{-0.5x}$$

$x \to \boxed{\times 0.5} \to \boxed{\times(-1)} \to \boxed{f} \to g$

- reflect over y
- h. stretch by 2



Parent	$f(x)=2^x$	$f(x)=2^x$
Transformed function $g(x)$	$g(x)=2^{2x+6}$	$g(x)=2^{2(x+3)}$
$g(x)$in terms of $f(x)$	$g(x)=f(2x+6)$	$g(x)=f[2(x+3)]$
Input Output Diagram	$x\to\boxed{\times 2}\to\boxed{+6}\to\boxed{f}\to g$	$x\to\boxed{+3}\to\boxed{\times 2}\to\boxed{f}\to g$
Description of transformations (in the correct order)	1 - left 6 2 - h. compress by 2	1 h. compress by 2 2 left 3
Graph		
Key Features	Asymptote: $y=0$ Increasing/ Decreasing: inc Y-intercept: $2^6=64$ Growth/Decay: grow	Asymptote: $y=0$ Increasing/ Decreasing: inc Y-intercept: 64 Growth/Decay: grow

ALRIGHT STOP!! THE TWO GRAPHS ABOVE SHOULD BE IDENTICAL. IF THEY ARE NOT, THERE'S A MISTAKE. NOTE THE TWO FORMATS:

$$g(x)=2^{2x+6} \quad vs \quad g(x)=2^{2(x+3)}$$

NOTE THE TRANSFORMATIONS ASSOCIATED WITH BOTH, AND PAY PARTICULAR ATTENTION TO THE ORDER. THE TRANSFORMATIONS ARE DIFFERENT AND THE ORDER IS DIFFERENT, BUT THE RESULT IS THE SAME.

Identify the transformations (in correct order) of the following. Then common factor the exponent, and identify the transformations (in correct order).

$g(x)=\left(\frac{1}{2}\right)^{3x-12}$

$x\to\boxed{\times 3}\to\boxed{-12}\to\boxed{f}\to g$

- right 12
- h. compress by 3

$g(x)=\left(\frac{1}{2}\right)^{3(x-4)}$

$x\to\boxed{-4}\to\boxed{\times 3}\to\boxed{f}\to g$

- h. compress by 3
- right 4

Graph each of the following.

$g(x) = -2^x + 4$

$= -(2^x) + 4$

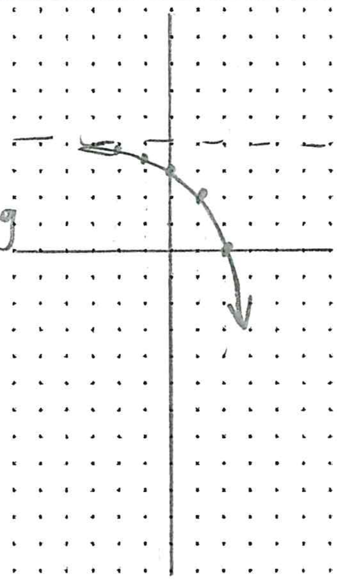
Transformations:
(in order):

$x \to \boxed{F} \to \boxed{x(-1)} \to \boxed{+4} \to g$

- reflect over x
- up 4

Asymptote: 4

Y-intercept: 3



$g(x) = 2^{-x} + 3$

Transformations:
(in order):

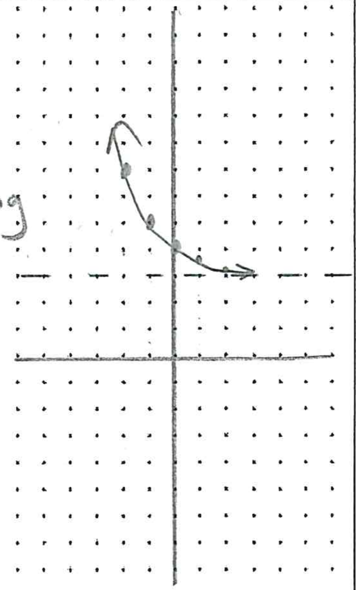
$x \to \boxed{x(-1)} \to \boxed{F} \to \boxed{+3} \to g$

-reflect over y

-up 3

Asymptote: $Y=3$

Y-intercept:
4



$g(x) = 2(2)^{x-4}$

Transformations:
(in order):

$x \to \boxed{-4} \to \boxed{F} \to \boxed{\times 2} \to g$

-v. stretch by 2

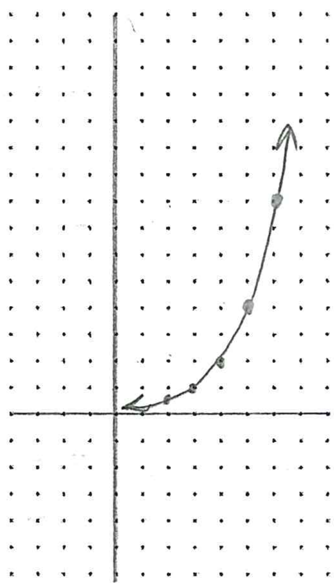
-right 4

Asymptote: $y=0$

Y-intercept:

$2(2)^{-4} = 2\left(\frac{1}{16}\right)$

$= \frac{1}{8}$



$h(x) = -\frac{1}{4}(2)^x$

Transformations:
(in order):

$x \to \boxed{F} \to \boxed{\times \frac{1}{4}} \to \boxed{x(-1)} \to g$

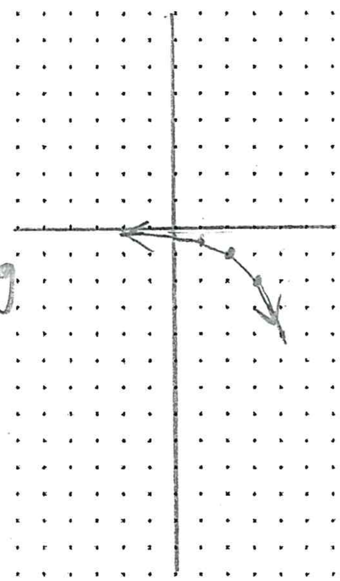
-V. compress by 4

-reflect over x

Asymptote: $Y=0$

Y-intercept:

$-\frac{1}{4}$



$g(x) = 2^{2(x-4)}$

Transformations:
(in order):

$x \to \boxed{-4} \to \boxed{\times 2} \to \boxed{F} \to g$

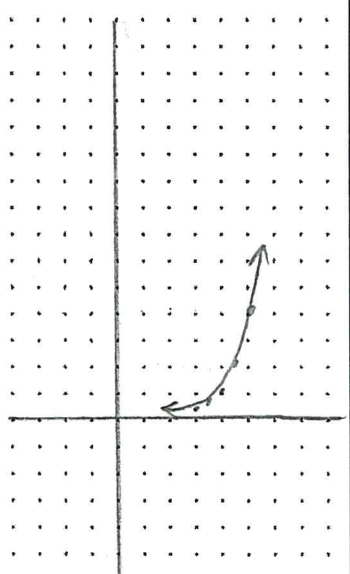
-h. compress by 2

-right 4

Asymptote: $Y=0$

Y-intercept:

$2^{-8} = \frac{1}{256}$



$b(x) = 2^{\frac{1}{2}x+2}$

Transformations:
(in order):

$x \to \boxed{x \frac{1}{2}} \to \boxed{+2} \to \boxed{F} \to g$

-left 2

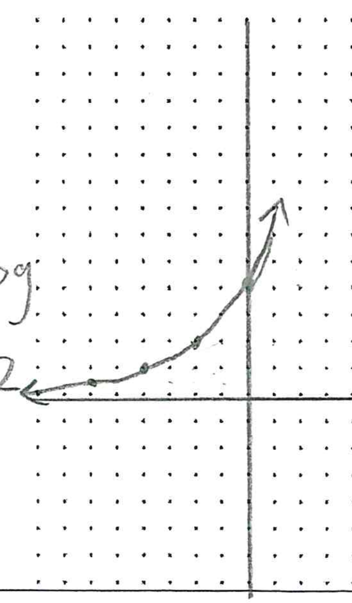
-h. stretch by 2

Asymptote:

$Y=0$

Y-intercept:

$Y=4$



-6- $x \to \boxed{\times 2} \to \boxed{+4} \to \boxed{f} \to \boxed{-2} \to g$

Graph each of the following.

$g(x) = 0.5(3)^{-x} + 1$

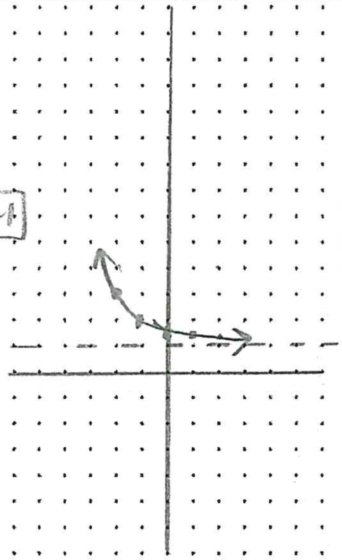
Transformations:
(in order):

$x \to \boxed{\times(-1)} \to \boxed{f} \to \boxed{\times 0.5} \to \boxed{+1} \to g$

- reflect over y
- v. compress by 2
- up 1

Asymptote: $y=1$

Y-intercept: 1.5



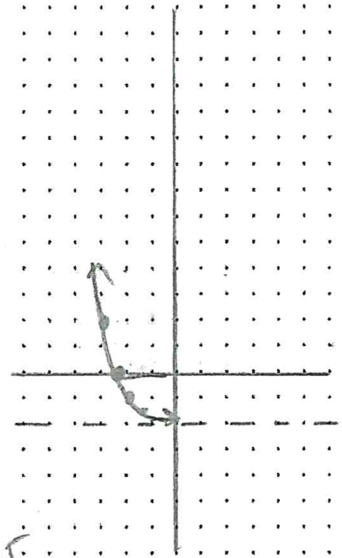
$y = \left(\frac{1}{2}\right)^{2x+4} - 2$

Transformations:
(in order):

- left 4
- h. compress by 2
- down 2

Asymptote: $y = -2$

Y-intercept:
$\left(\frac{1}{2}\right)^4 - 2 = \frac{1}{16} - 2$
≈ -1.9375



1) Start with the function $f(x) = 3^x$. Build an input-output diagram and write the equation of a transformed function having undergone the following transformations (in order):

a) vertical stretch by 2, translation down 3

$x \to \boxed{f} \to \boxed{\times 2} \to \boxed{-3} \to g$ $\boxed{g(x) = 2(3^x) - 3}$

b) reflection over the y axis, left 4

$x \to \boxed{+4} \to \boxed{x(-1)} \to \boxed{f} \to g$ $\boxed{g(x) = 3^{-(x+4)}}$

c) reflection over x, translation up 2, horizontal stretch by 2, translation left 3

y x

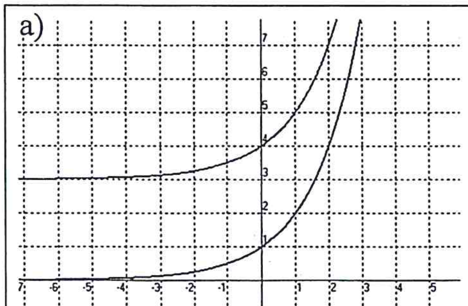
$x \to \boxed{+3} \to \boxed{\times \frac{1}{2}} \to \boxed{f} \to \boxed{\times(-1)} \to \boxed{+2} \to g$ $\boxed{g(x) = -3^{\frac{1}{2}(x+3)} + 2}$

d) translation right 3, horizontal compression by 2, vertical compression by 4

$x \to \boxed{\times 2} \to \boxed{-3} \to \boxed{f} \to \boxed{\times \frac{1}{4}} \to g$ $\boxed{g(x) = \frac{1}{4}(3^{2x-3})}$

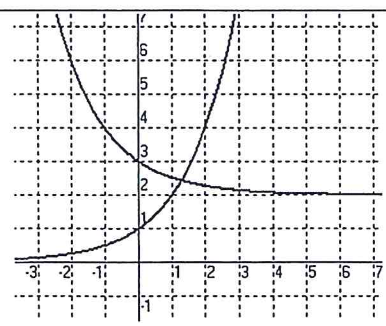
2) Determine the equation of the transformed function if the base function is $f(x)=2^x$.

a)



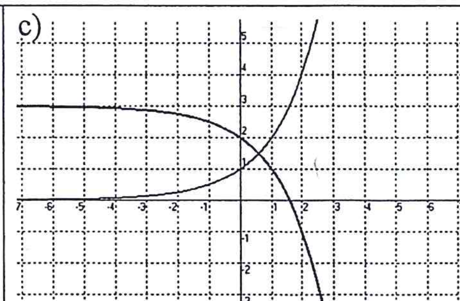
$g(x) = 2^x + 3$

b)



$g(x) = 2^{-x} + 2$

c)



$g(x) = -2^x + 3$