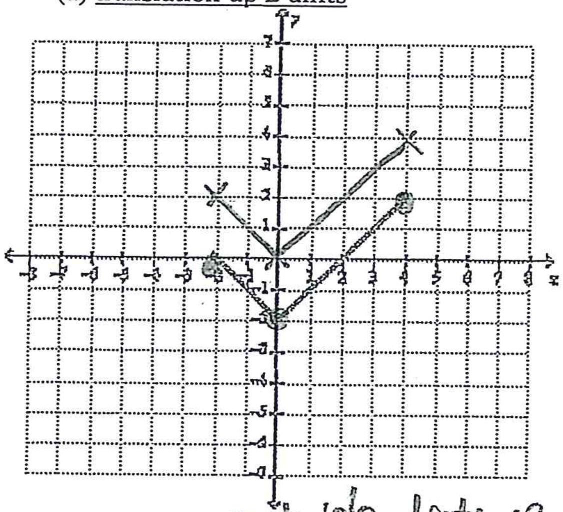
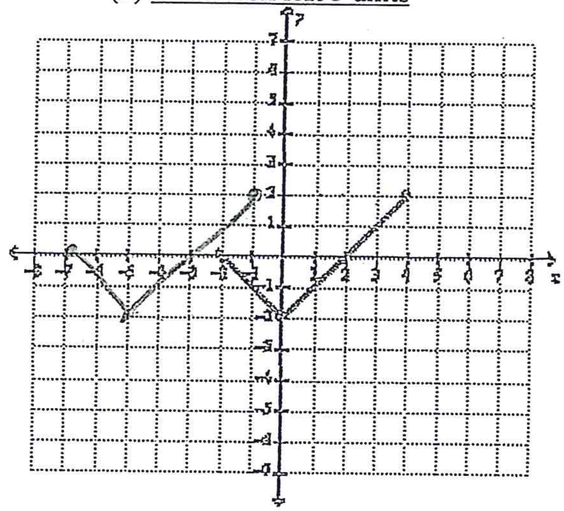
CR3U **Linear Piecewise Transformations** SOLUTIONS

The base function $f(x)$, shown below, is being transformed in one or more ways. Graph the transformed function by finding the new location of the three key points.

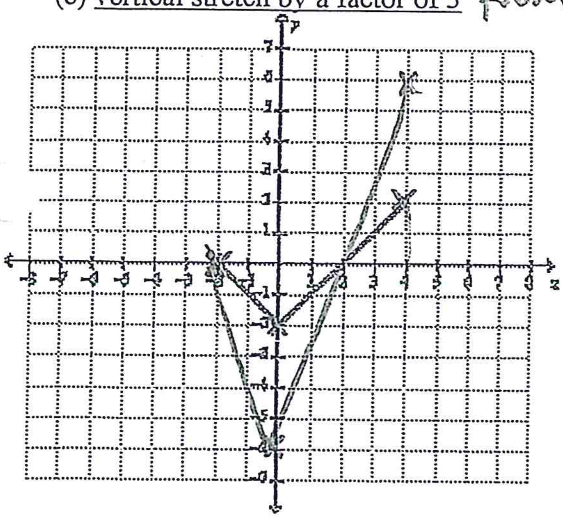
(a) translation up 2 units



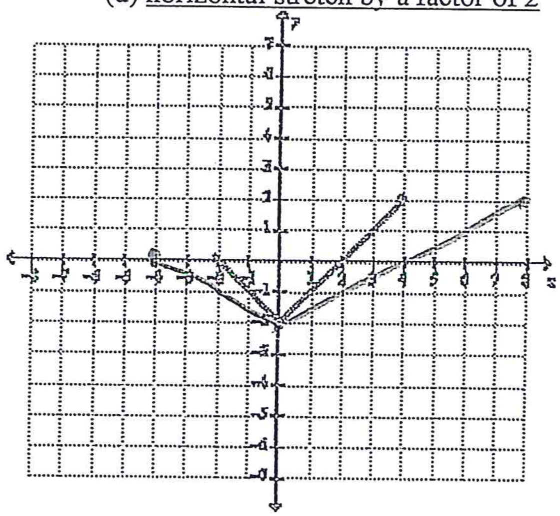
(b) translation left 5 units



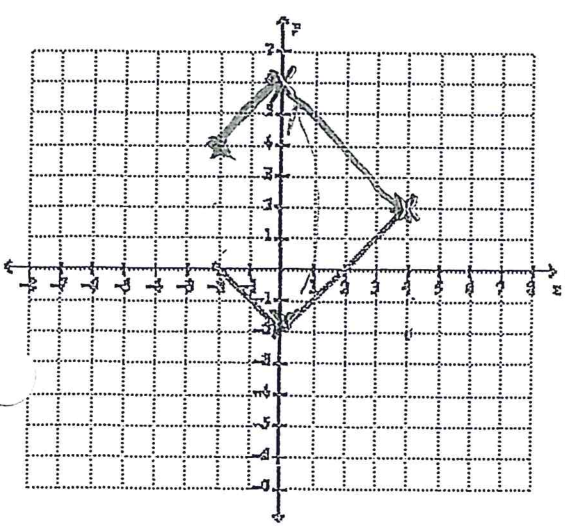
(c) vertical stretch by a factor of 3 → triple distance from x axis



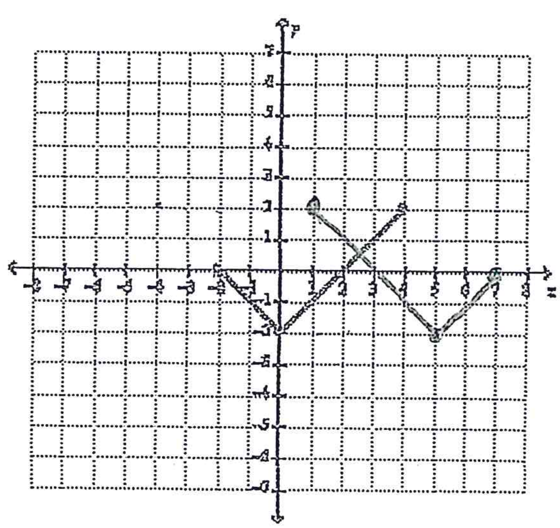
(d) horizontal stretch by a factor of 2



(e) reflection over the x-axis and a translation up 4 units



f) reflection over the y-axis and a translation right 5 units



a) $x \to \boxed{\times 3} \to \boxed{+4} \to f(x)$

-4	-12	-8
0	0	4
3	9	13
8	24	28

b) $x \to \boxed{\times 0.5} \to \boxed{-3} \to f(x)$

-4	-2	-5
0	0	-3
3	1.5	-1.5
8	4	1

c) $x \to \boxed{\times -2} \to \boxed{-4} \to r(x)$

-4	8	4
0	0	-4
3	-6	-10
8	-16	-20

d) $x \to \boxed{(\ )^2} \to \boxed{+2} \to f(x)$

-4	16	18
0	0	2
3	9	11
8	64	66

e) $x \to \boxed{(\ )^2} \to \boxed{\times 0.5} \to g(x)$

-4	16	8
0	0	0
3	9	4.5
8	64	32

f) $x \to \boxed{-3} \to \boxed{(\ )^2} \to h(x)$

-4	-7	49
0	-3	9
3	0	0
8	5	25

g) $x \to \boxed{-4} \to \boxed{(\ )^2} \to \boxed{\times 2} \to h(x)$

-4	-8	64	128
0	-4	16	32
3	-1	1	2
8	4	16	32

h) $x \to \boxed{(\ )^2} \to \boxed{\times(-1)} \to \boxed{-5} \to k(x)$

-4	16	-16	-21
0	0	0	-5
3	9	-9	-14
8	64	-64	-69

i) $x \to \boxed{+2} \to \boxed{(\ )^2} \to \boxed{\times 3} \to \boxed{-5} \to f(x)$

-4	-2	4	12	7
0	2	4	12	7
3	5	25	75	70
8	10	100	300	295

j) $x \to \boxed{-1} \to \boxed{(\ )^2} \to \boxed{\times(-1)} \to p(x)$

-4	-5	25	-25
0	-1	1	-1
3	2	4	-4
8	7	49	-49

Building and Using Input Output Diagrams USE A SEPARATE SHEET OF PAPER AS NEEDED

1. In each case build an input output-diagram and use it to find the outputs for x=-4, x=0, x=3, x=8

a) $f(x)=3x+4$	b) $f(x)=\frac{1}{2}x-3$
c) $r(x)=-2x-4$	d) $f(x)=x^2+2$
e) $g(x)=0.5x^2$	f) $f(x)=(x-3)^2$
g) $h(x)=2(x-4)^2$	h) $k(x)=-x^2-5$
i) $f(x)=3(x+2)^2-5$	j) $p(x)=-(x-1)^2$

2. Given the input-output diagram, find the inputs for the given outputs.
Then write an equation that is represented by the input output diagram.

a)

$$X \longrightarrow \boxed{\text{x2}} \longrightarrow \boxed{-1} \longrightarrow f(x)$$

3	6	5
0	0	-1
2.5	5	4
4.5	9	8

Equation: $f(x)=2x-1$

b)

$$X \longrightarrow \boxed{\text{x}(-1)} \longrightarrow \boxed{+3} \longrightarrow g(x)$$

-2	2	5
4	-4	-1
-1	1	4
-5	5	8

Equation: $g(x)=-x+3$

c)

$$X \longrightarrow \boxed{(\,)^2} \longrightarrow \boxed{+2} \longrightarrow h(x)$$

3	9	11
2	4	6
1	1	3
	-4	-2

Equation: $h(x)=x^2+2$

d)

$$X \longrightarrow \boxed{-3} \longrightarrow \boxed{(\,)^2} \longrightarrow h(x)$$

7	4	16
6	3	9
5	2	4
4	1	1

Equation: $h(x)=(x-3)^2$

e)

$$X \longrightarrow \boxed{(\,)^2} \longrightarrow \boxed{\text{x}(-1)} \longrightarrow \boxed{+1} \longrightarrow t(x)$$

0	0	0	1
1	1	-1	0
2	4	-4	-3
3	9	-9	-8

Equation: $t(x)=-x^2+1$

f)

$$X \longrightarrow \boxed{+3} \longrightarrow \boxed{(\,)^2} \longrightarrow \boxed{+4} \longrightarrow p(x)$$

-1	2	4	8
0	3	9	13
1	4	16	20
2	5	25	29

Equation: $p(x)=(x+3)^2+4$

3. Complete the values of the input output diagram by working outwards. Write an equation.

a)

$x \rightarrow \boxed{-4} \rightarrow \boxed{(\ )^2} \rightarrow \boxed{\times 2} \rightarrow p(x)$

2	-2	4	8
3	-1	1	2
4	0	0	0
5	1	1	2
6	2	4	8

Equation: $p(x)=2(x-4)^2$

b)

$x \rightarrow \boxed{+5} \rightarrow \boxed{(\ )^2} \rightarrow \boxed{-2} \rightarrow p(x)$

-7	-2	4	2
-6	-1	1	-1
-5	0	0	-2
-4	1	1	-1
-3	2	4	2

Equation: $p(x)=(x+5)^2-2$

c)

$x \rightarrow \boxed{\times 0.5} \rightarrow \boxed{(\ )^2} \rightarrow \boxed{\times(-1)} \rightarrow t(x)$

-4	-2	4	-4
-2	-1	1	-1
0	0	0	0
2	1	1	-1
4	2	4	-4

Equation: $t(x)=-(0.5x)^2$

d)

$x \rightarrow \boxed{+1} \rightarrow \boxed{(\ )^2} \rightarrow \boxed{\times(-1)} \rightarrow \boxed{+5} \rightarrow f(x)$

-3	-2	4	-4	1
-2	-1	1	-1	4
-1	0	0	0	5
0	1	1	-1	4
1	2	4	-4	1

Equation: $f(x)=-(x+1)^2+5$

4. Given the values, complete the input-output diagram. Write an equation to match.

a)

$x \rightarrow \boxed{-4} \rightarrow \boxed{(\ )^2} \rightarrow \boxed{+4} \rightarrow p(x)$

3	-1	1	5
4	0	0	4
5	1	1	5
6	2	4	8
7	3	9	13

Equation: $p(x)=$

b)

$x \rightarrow \boxed{+5} \rightarrow \boxed{(\ )^2} \rightarrow \boxed{\times(-2)} \rightarrow p(x)$

-7	-2	4	-8
-6	-1	1	-2
-5	0	0	0
-4	1	1	-2
-3	2	4	-8

Equation: $p(x)=-2(x+5)^2$

c)

$x \rightarrow \boxed{\sqrt{\ }} \rightarrow \boxed{\times(-1)} \rightarrow \boxed{-2} \rightarrow p(x)$

1	1	-1	-3
4	2	-2	-4
9	3	-3	-5
16	4	-4	-6

Equation: $p(x)=-\sqrt{x}-2$

d)

$x \rightarrow \boxed{\times(-1)} \rightarrow \boxed{-3} \rightarrow \boxed{\sqrt{\ }} \rightarrow p(x)$

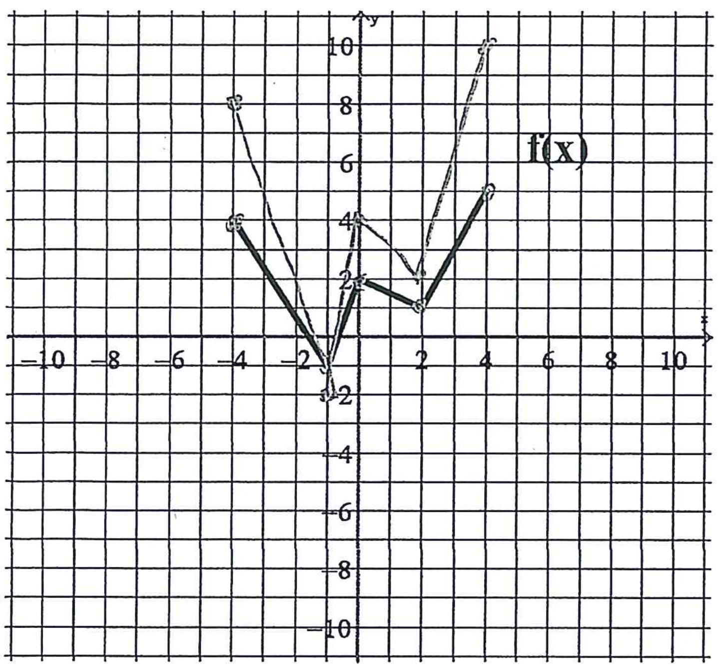
-3	3	0	0
-4	4	1	1
-7	7	4	2
-12	12	9	3

Equation: $p(x)=\sqrt{-x-3}$

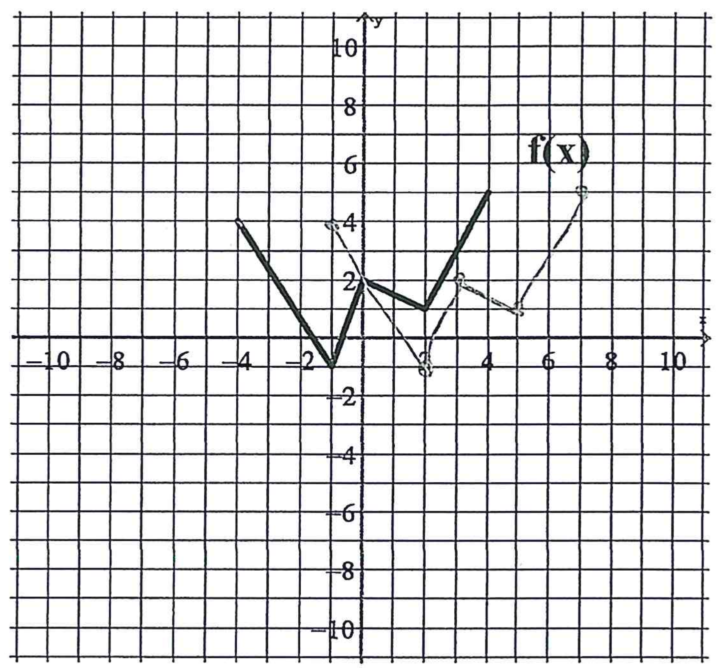
MCR3U **Linear Piecewise Transformations**

The function f(x) is shown below. In each case it is being transformed to give a new function, g(x). Start by identifying the key points of f(x). Then, using a separate paper, in each case:
a) Build an input/output diagram for g(x) using the equation, and add the key points of f(x)
b) Work outwards in your input output diagram to identify inputs and outputs of g(x)
c) Graph the transformed function g(x) using the points from your input-output diagram
d) Identify the transformations applied to f(x) to result in g(x). Use your graph or I/O diagram.

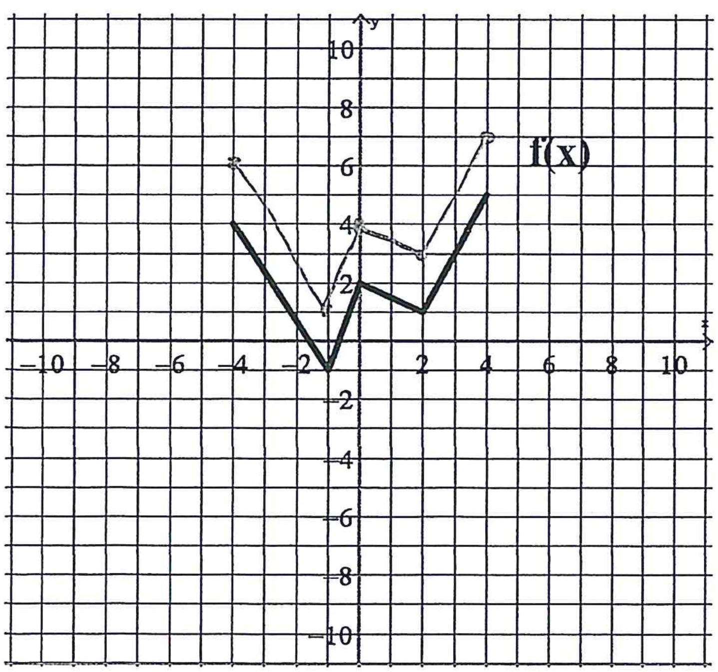
1. $g(x)=2f(x)$ → v. stretch by 2



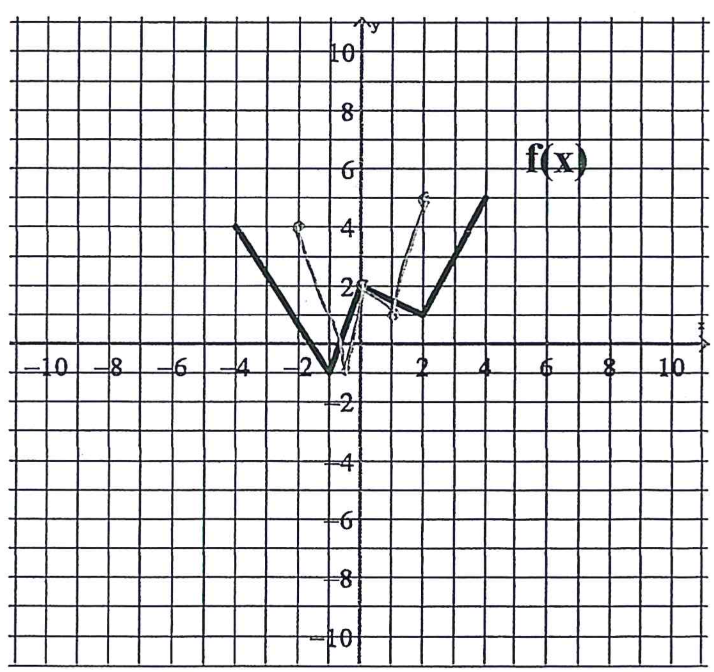
2. $g(x)=f(x-3)$ → ~~left~~ right 3



3. $g(x)=f(x)+2$ → up 2

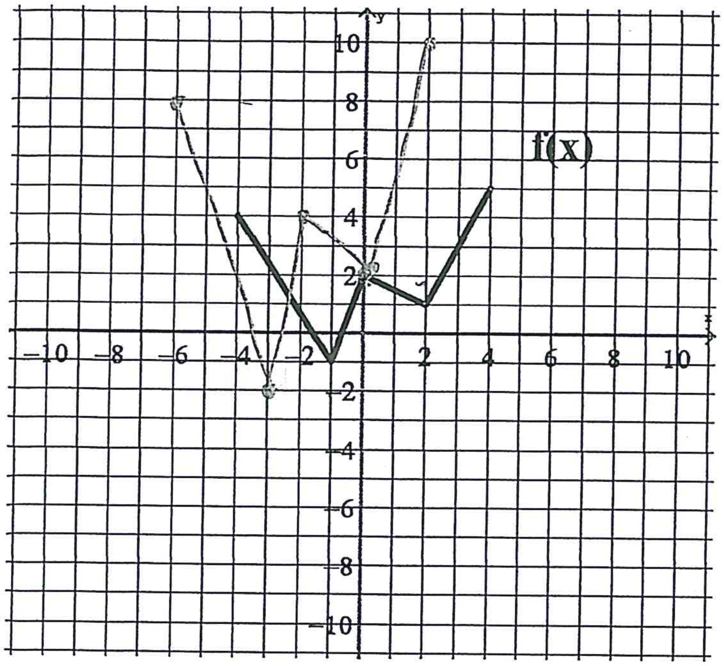


4. $g(x)=f(2x)$ → h. compress by 2



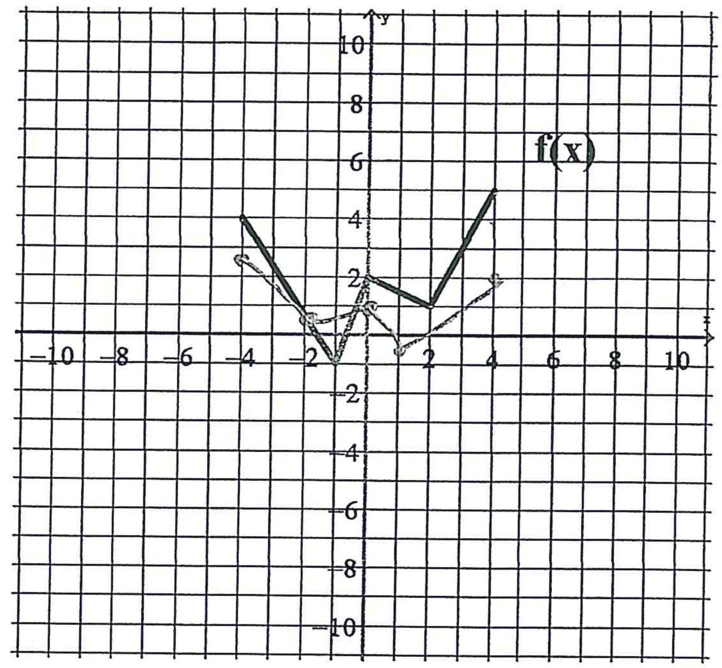
v. stretch by 2 → left 2

5. $g(x)=2f(x+2)$



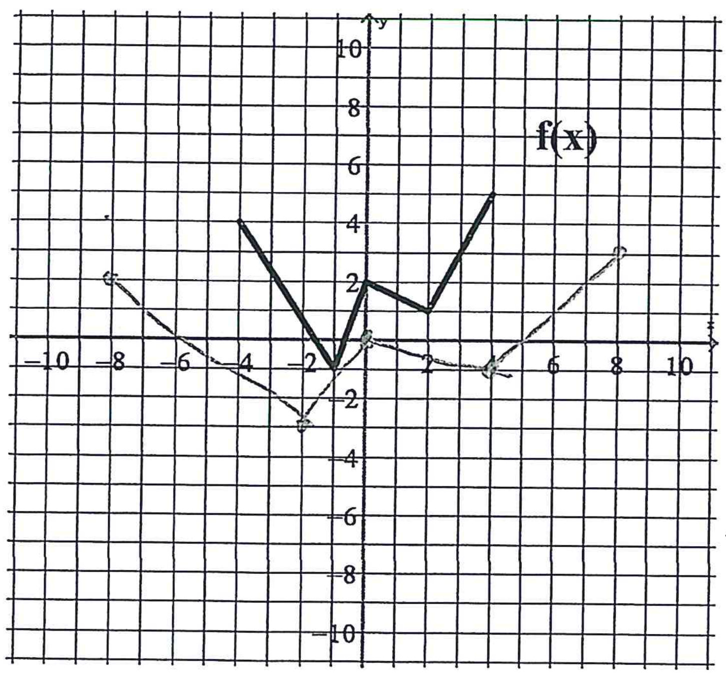
v. compress by 2 → reflect over y

6. $g(x)=\frac{1}{2}f(-x)$



h-stretch by 2 → down 2

7. $g(x)=f\left(\frac{1}{2}x\right)-2$

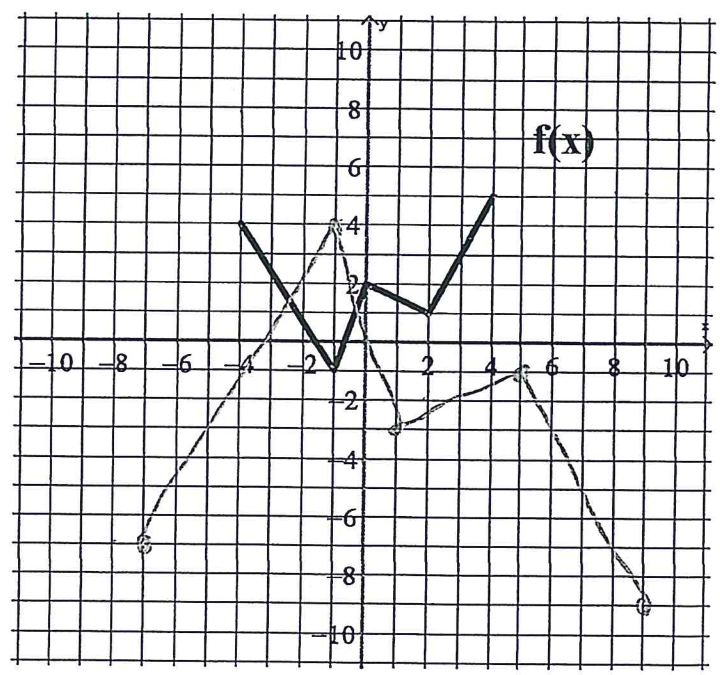


v. stretch by 2 and reflect over x

h. stretch by 2

right 1

8. $g(x)=-2f(0.5(x-1))+1$ → up 1



i. key points of $f(x)$:

x	$f(x)$
-4	4
-1	-1
0	2
2	1
4	5

1. $g(x) = 2f(x)$

$x \to \boxed{f} \to \boxed{\times 2} \to g(x)$

-4	4	8
-1	-1	-2
0	2	4
2	1	2
4	5	10
x		y

v. stretch by 2

2. $g(x) = f(x-3)$

$x \to \boxed{-3} \to \boxed{f} \to g(x)$

-1	-4	4
2	-1	-1
3	0	2
5	2	1
7	4	5

left 3

3. $g(x) = f(x) + 2$

$x \to \boxed{f} \to \boxed{+2} \to g(x)$

-4	4	6
-1	-1	1
0	2	4
2	1	3
4	5	7

up 2

4. $g(x) = f(2x)$

$x \to \boxed{\times 2} \to \boxed{f} \to g(x)$

-2	-4	4
-0.5	-1	-1
0	0	2
1	2	1
2	4	5

h. compress by 2

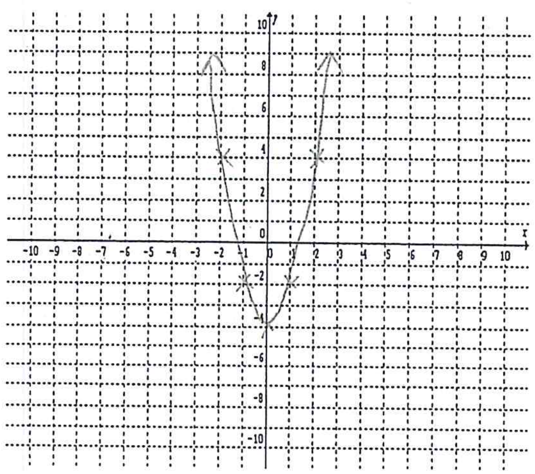
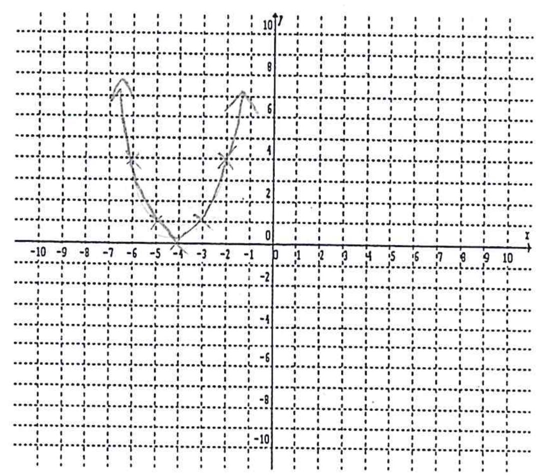
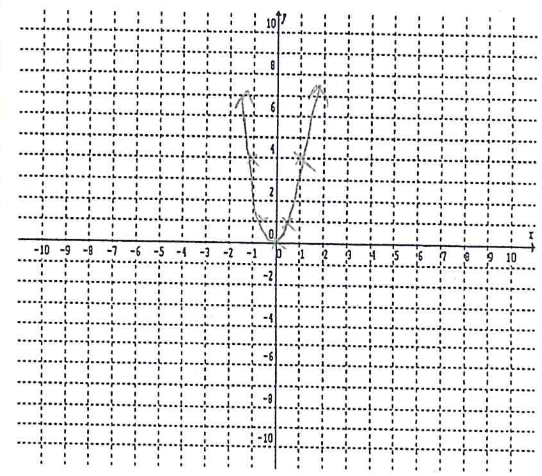
Using Function Notation With a Variety of Function Types

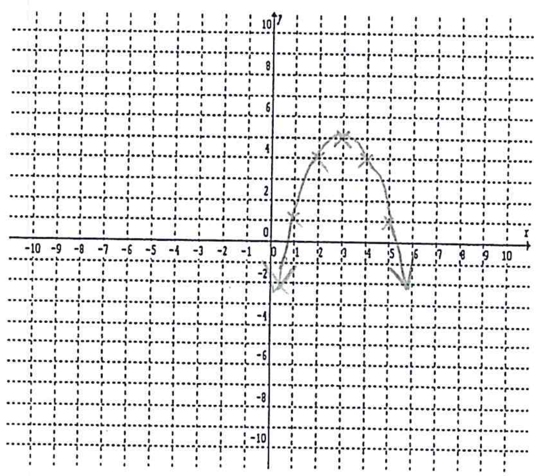
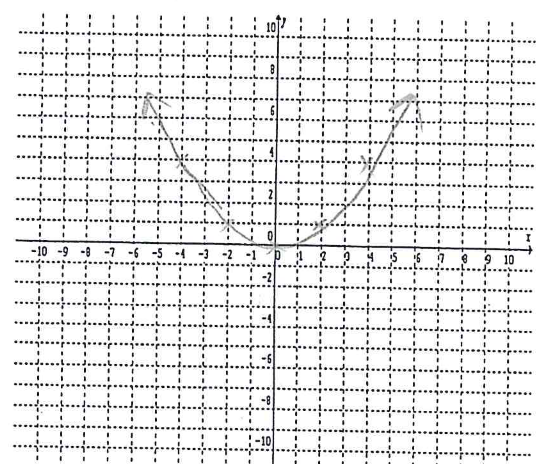
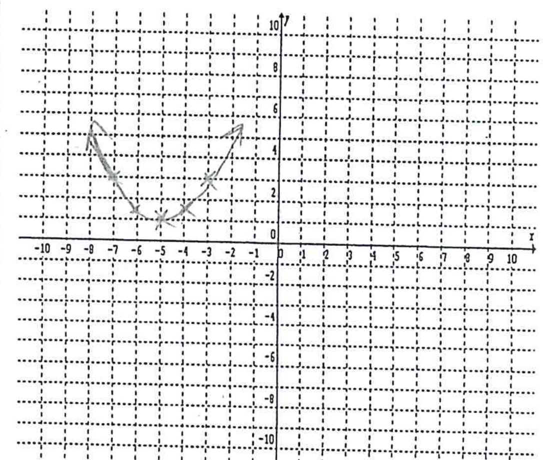
Complete the table

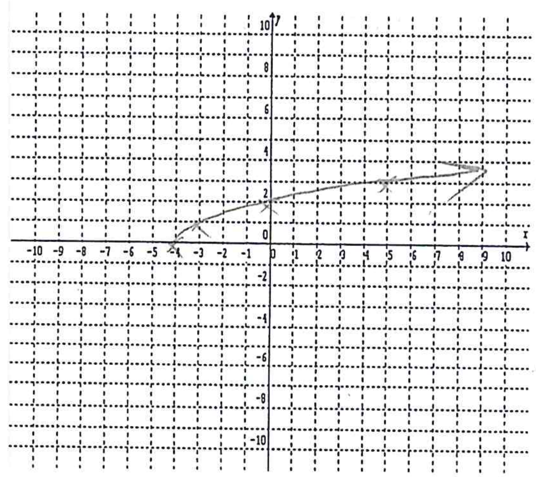
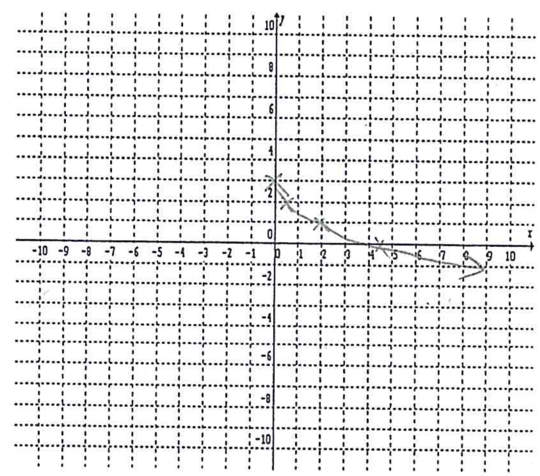
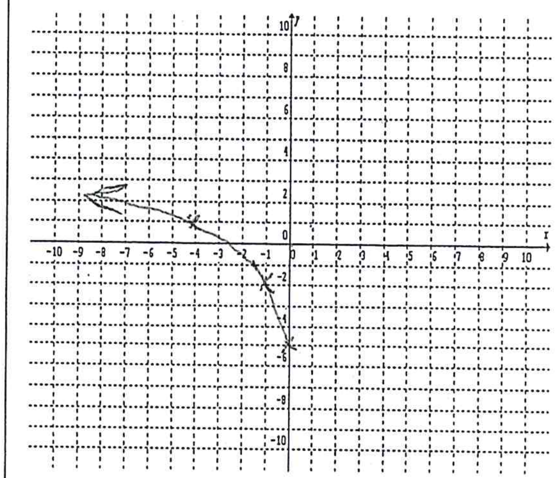
	Transformed Function if Parent Function is:		
Transformed Function in terms of $f(x)$	$f(x)=x^2$	$f(x)=\sqrt{x}$	$f(x)=\sin x$
$g(x)=f(x)+3$	$g(x)=x^2+3$	$g(x)=\sqrt{x}+3$	$g(x)=\sin x+3$
$g(x)=f(x)-5$	$g(x)=x^2-5$	$g(x)=\sqrt{x}-5$	$g(x)=\sin x-5$
$g(x)=f(x-3)$	$g(x)=(x-3)^2$	$g(x)=\sqrt{x-3}$	$g(x)=\sin(x-3)$
$g(x)=f(x+4)$	$g(x)=(x+4)^2$	$g(x)=\sqrt{x+4}$	$g(x)=\sin(x+4)$
$g(x)=f(x+2)-1$	$g(x)=(x+2)^2-1$	$g(x)=\sqrt{x+2}-1$	$g(x)=\sin(x+2)-1$
$g(x)=f(x-5)-8$	$g(x)=(x-5)^2-8$	$g(x)=\sqrt{x-5}-8$	$g(x)=\sin(x-5)-8$
$g(x)=2f(x-3)$	$g(x)=2(x-3)^2$	$g(x)=2\sqrt{x-3}$	$g(x)=2\sin(x-3)$
$g(x)=f(-x)$	$g(x)=(-x)^2$	$g(x)=\sqrt{-x}$	$g(x)=\sin(-x)$
$g(x)=-f(x)+3$	$g(x)=-x^2+3$	$g(x)=-\sqrt{x}+3$	$g(x)=-\sin(x)+3$
$g(x)=-f(2x)$	$g(x)=-(2x)^2$	$g(x)=-\sqrt{2x}$	$g(x)=-\sin(2x)$
$g(x)=0.5f(2x)-1$	$g(x)=0.5(2x)^2-1$	$g(x)=0.5\sqrt{2x}-1$	$g(x)=0.5\sin(2x)-1$
$g(x)=f(x-4)+3$	$g(x)=(x-4)+3$	$g(x)=\sqrt{x-4}+3$	$g(x)=\sin(x-4)+3$
$g(x)=f(-0.5x)$	$g(x)=(-0.5x)^2$	$g(x)=\sqrt{-0.5x}$	$g(x)=\sin(-0.5x)$

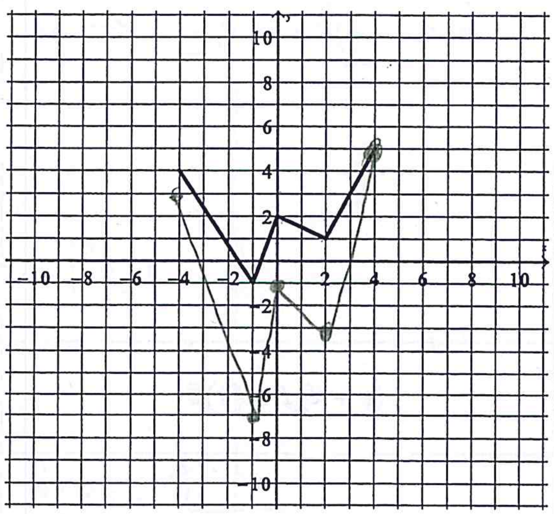
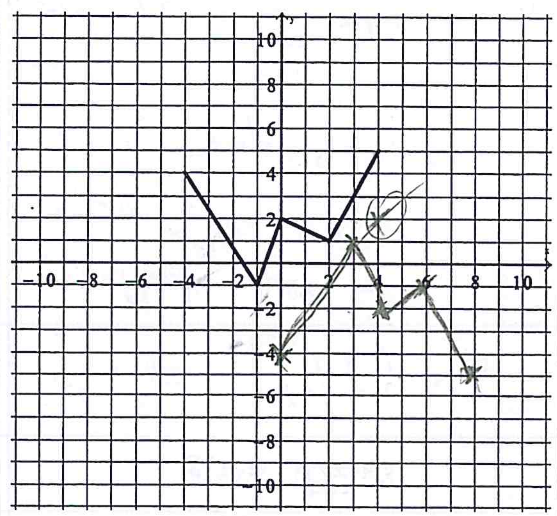
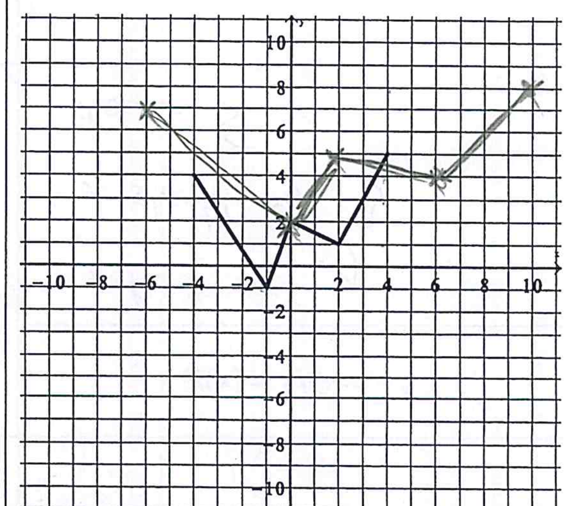
Different representations of transfo.. ..ations – Quadratic, Root and Piecewise Functions

Solns.

Parent function	$f(x)=x^2$	$f(x)=x^2$	$f(x)=x^2$
Transformed function $g(x)$	$g(x)=2x^2-4$	$g(x)=(x+4)^2$	$f(x)=(2x)^2$
Transformed function $g(x)$ in terms of $f(x)$	$g(x)=2f(x)-4$	$g(x)=f(x+4)$	$g(x)=f(2x)$
Description of transformations	– v. stretch by 2 – down 4	– left 4	– h. compress by 2
Input Output Diagram	$x \to \boxed{(\ )^2} \to \boxed{\times 2} \to \boxed{-4} \to g(x)$	$x \to \boxed{+4} \to \boxed{f} \to g(x)$ -2 4 -1 1 0 0 1 1 2 4	$x \to \boxed{\times 2} \to \boxed{f} \to g$
Transformation of the point (x, y)			
Graph			

Parent Function	$f(x)=x^2$	$f(x)=x^2$	$f(x)=x^2$
Transformed function $g(x)$	$g(x) = -(x-3)^2 + 5$	$g(x)=(-\frac{1}{2}x)^2$	$g(x)=\frac{1}{2}(x+5)^2+1$
Transformed function $g(x)$ in terms of $f(x)$	$g(x)=-f(x-3)+5$	$g(x) = f(-\frac{1}{2}x)$	$g(x) = \frac{1}{2}f(x+5)+1$
Description of transformations	- reflection over x - up 5 - right 3	- horizontal stretch by 2 - reflection over y	- v. compress by 2 - up 1 - left 5
Input Output Diagram	$x \to \boxed{-3} \to \boxed{f} \to \boxed{x(-1)} \to \boxed{+5} \to g$	$x \to \boxed{x\,\frac{1}{2}} \to \boxed{x(-1)} \to \boxed{f} \to g$ OR $x \to \boxed{x(-1)} \to \boxed{x\,\frac{1}{2}} \to \boxed{f} \to g$ OR $x \to \boxed{x\,\frac{1}{2}} \to \boxed{f} \to g$	$x \to \boxed{+5} \to \boxed{f} \to \boxed{x\,\frac{1}{2}} \to \boxed{+1} \to g$
Transformation of the point (x, y)			
Graph			

Parent function	$f(x)=\sqrt{x}$	$f(x)=\sqrt{x}$	$f(x)=\sqrt{x}$
Transformed function $g(x)$	$g(x)=\sqrt{x+4}$	$g(x)=-\sqrt{2x}+3$	$g(x)=3\sqrt{-x}-5$
Transformed function $g(x)$ in terms of $f(x)$	$g(x)=f(x+4)$	$g(x)=-f(2x)+3$	$g(x)=3f(-x)-5$
Description of transformations	left 4	-reflect over x -up 3 - h. compress by 2	- reflect over y -v. stretch by 3 -down 5
Input Output Diagram	$x \rightarrow \boxed{+4} \rightarrow \boxed{f} \rightarrow g$	$x \rightarrow \boxed{\times 2} \rightarrow \boxed{f} \rightarrow \boxed{\times(-1)} \rightarrow \boxed{+3} \rightarrow g$	$x \rightarrow \boxed{\times(-1)} \rightarrow \boxed{f} \rightarrow \boxed{\times 3} \rightarrow \boxed{-5} \rightarrow g$
Transformation of the point (x, y)			
Graph			

Parent function	$f(x)$ in graph below	$f(x)$ in graph below	$f(x)$ in graph below
Transformed function $g(x)$ in terms of $f(x)$	$g(x) = 2f(x) - 5$	$g(x) = -f(x-4)$	$g(x) = f\left[\frac{1}{2}(x-2)\right] + 3$
Description of transformations	- v. stretch by 2 -down 5	- reflect over x - right 4	- up 3 - h. stretch by 2 - right 2
Input Output Diagram	$x \to \boxed{f} \to \boxed{\times 2} \to \boxed{-5} \to y$	$x \to \boxed{-4} \to \boxed{f} \to \boxed{\times(-1)} \to g$	$x \to \boxed{-2} \to \boxed{\times \frac{1}{2}} \to \boxed{f} \to \boxed{+3}$
Transformation of the point (x, y)			
Graph			

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