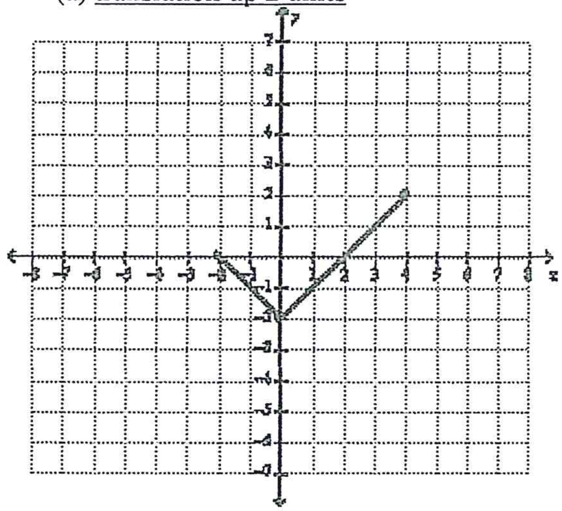
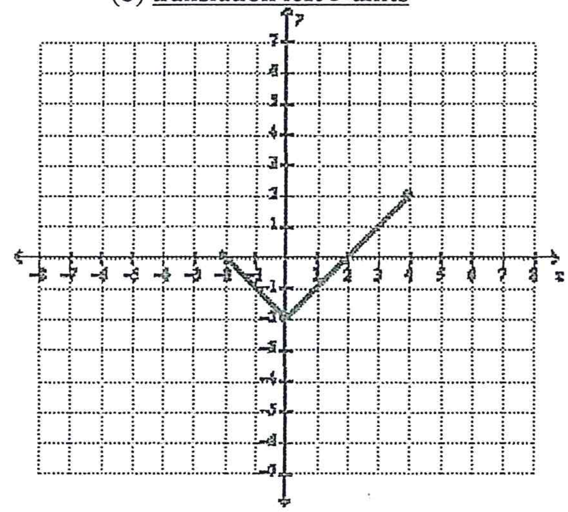
Linear Piecewise Transformations

The base function *f(x)*, shown below, is being transformed in one or more ways. Graph the transformed function by finding the new location of the three key points.

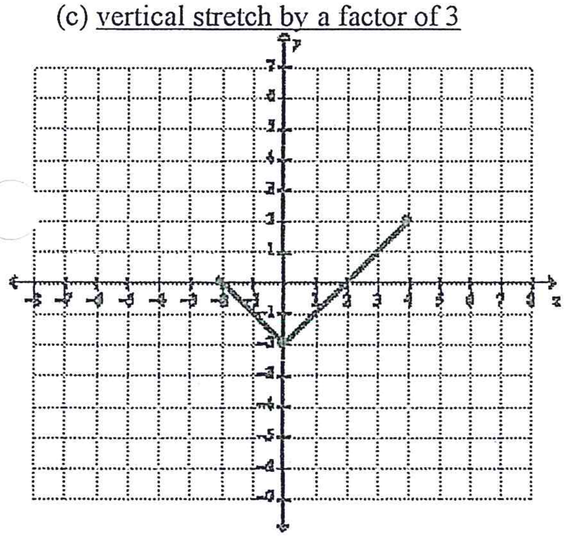
(a) translation up 2 units



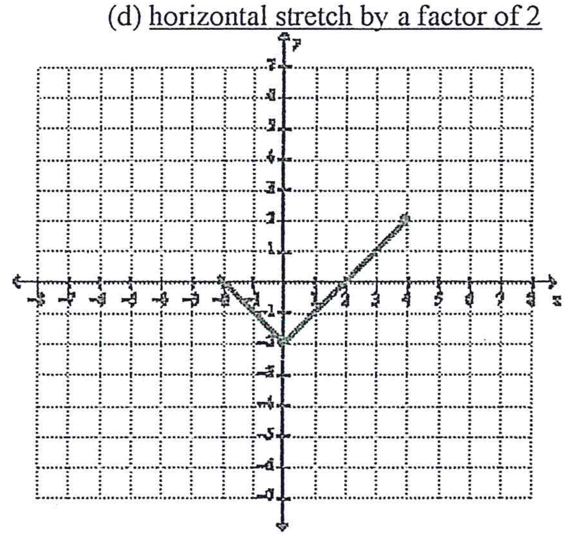
(b) translation left 5 units



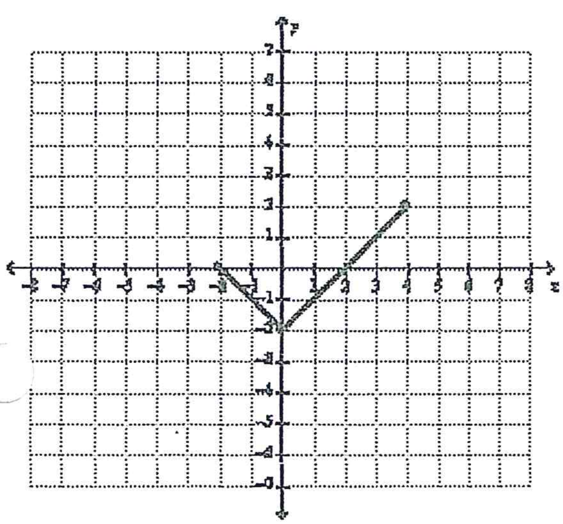
(c) vertical stretch by a factor of 3



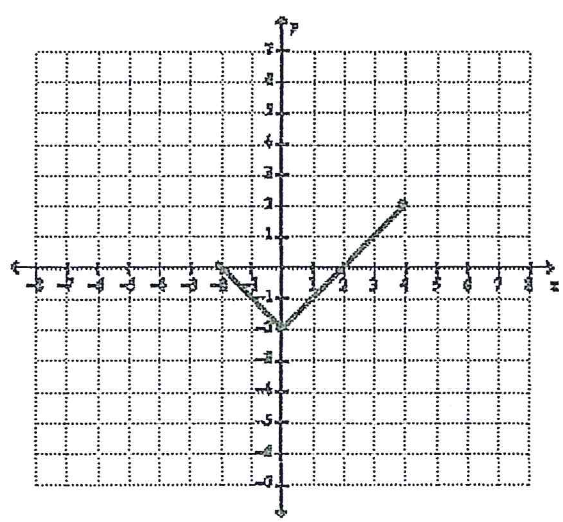
(d) horizontal stretch by a factor of 2



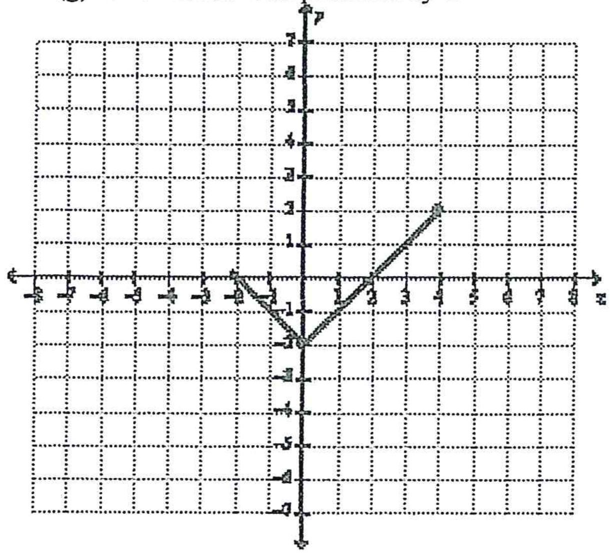
(e) reflection over the x-axis and a translation up 4 units



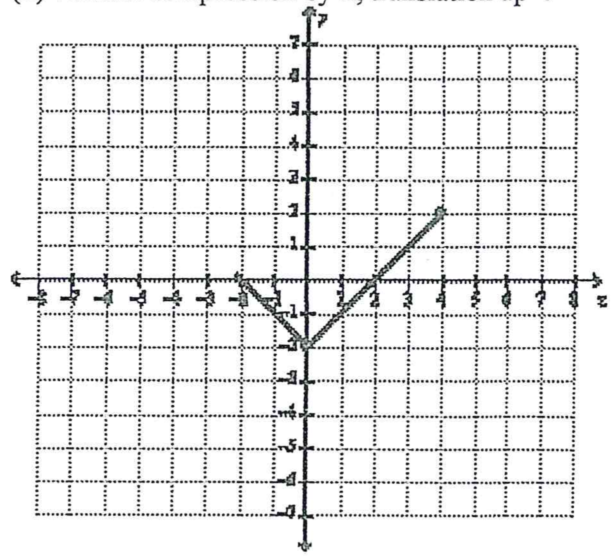
f) reflection over the y-axis and a translation right 5 units



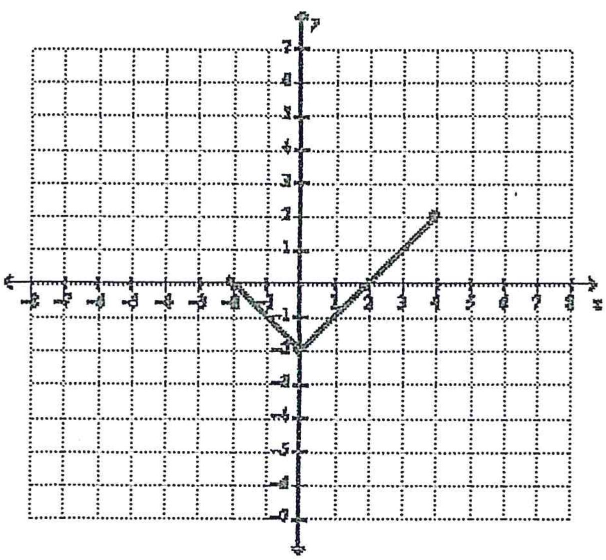
(g) a horizontal compression by 2



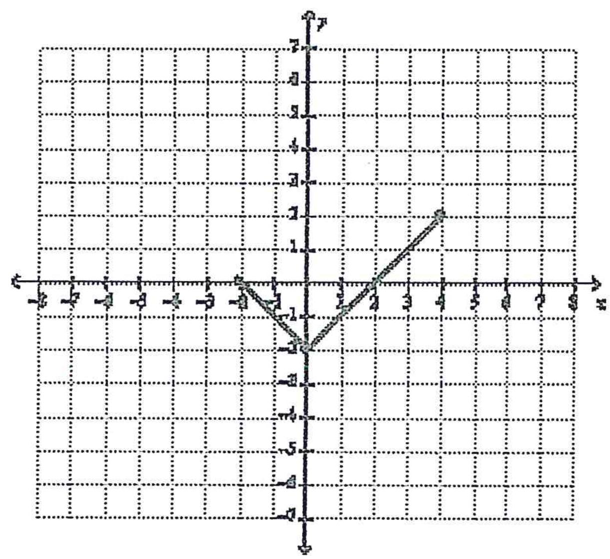
(h) vertical compression by 2, translation up 4



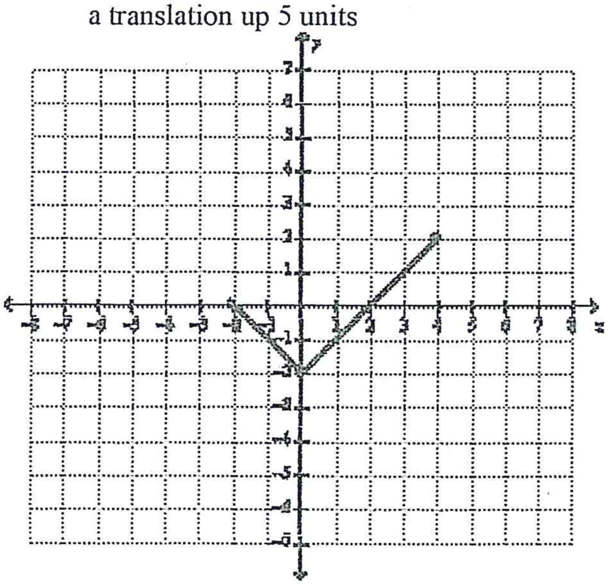
(i) reflection over the x-axis, translation down 3



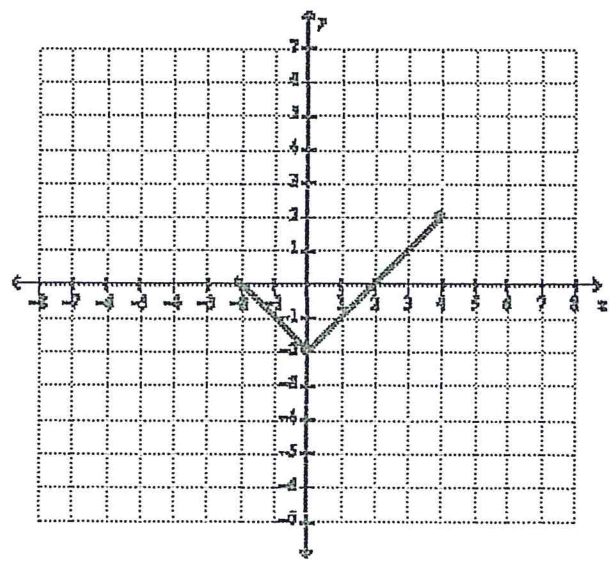
(j) a reflection over both axes



(k) a reflection over the y-axis,
a horizontal stretch by 2 and
a translation up 5 units



(l) a vertical stretch by 2, horizontal stretch by 2,
reflection over x, translation left 3



Building and Using Input Output Diagrams
USE A SEPARATE SHEET OF PAPER AS NEEDED

1. In each case build an input output-diagram and use it to find the outputs for x=-4, x=0, x=3, x=8

a) $f(x)=3x+4$	b) $f(x)=\frac{1}{2}x-3$
c) $r(x)=-2x-4$	d) $f(x)=x^2+2$
e) $g(x)=0.5x^2$	f) $f(x)=(x-3)^2$
g) $h(x)=2(x-4)^2$	h) $k(x)=-x^2-5$
i) $f(x)=3(x+2)^2-5$	j) $p(x)=-(x-1)^2$

2. Given the input-output diagram, find the inputs for the given outputs.
Then write an equation that is represented by the input output diagram.

a)

X → [x2] → [-1] → f(x)

5
-1
4
8

Equation: f(x)=

b)

X → [x(-1)] → [+3] → g(x)

5
-1
4
8

Equation: g(x) =

c)

X → [( )²] → [+2] → h(x)

11
6
3
-2

Equation: h(x)=

d)

X → [-3] → [( )²] → h(x)

16
9
4
1

Equation:

e)

X → [( )²] → [x(-1)] → [+1] → t(x)

1
0
-3
-8

Equation:

f)

X → [+3] → [( )²] → [+4] → p(x)

8
13
20
29

Equation: p(x)=

3. Complete the values of the input output diagram by working outwards. Write an equation.

a)

x → [-4] → [( )^2] → [x2] → p(x)

	-2	4	8
	-1	1	2
	0	0	
	1		
	2		

Equation:

b)

x → [+5] → [( )^2] → [-2] → p(x)

	-2		
	-1		
	0		
-4	1		
-3	2	4	

Equation:

c)

x → [x0.5] → [( )^2] → [x(-1)] → t(x)

	-2		
	-1		
	0		
	1		
	2		

Equation:

d)

x → [+1] → [( )^2] → [x(-1)] → [+5] → f(x)

	-2			
	-1			
	0			
	1			
	2			

Equation:

4. Given the values, complete the input-output diagram. Write an equation to match.

a)

x → [ ] → [ ] → [ ] → p(x)

3	-1	1	5
4	0	0	4
5	1	1	5
6	2	4	8
7	3	9	13

Equation:

b)

x → [ ] → [ ] → [ ] → p(x)

-7	-2	4	-8
-6	-1	1	-2
-5	0	0	0
-4	1	1	-2
-3	2	4	-8

Equation:

c)

x → [ ] → [ ] → [ ] → p(x)

1	1	-1	-3
4	2	-2	-4
9	3	-3	-5
16	4	-4	-6

Equation:

d)

x → [ ] → [ ] → [ ] → p(x)

-3	3	0	0
-4	4	1	1
-7	7	4	2
-12	12	9	3

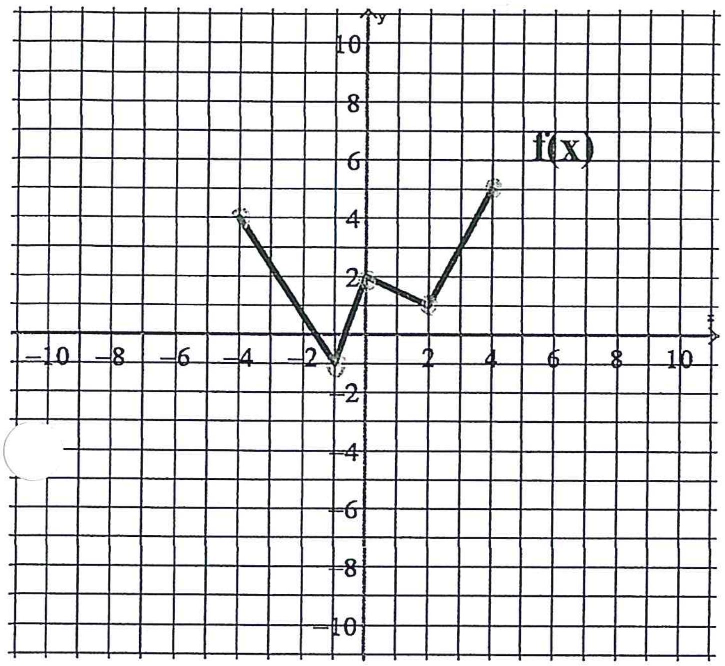
Equation:

MCR3U **Linear Piecewise Transformations**

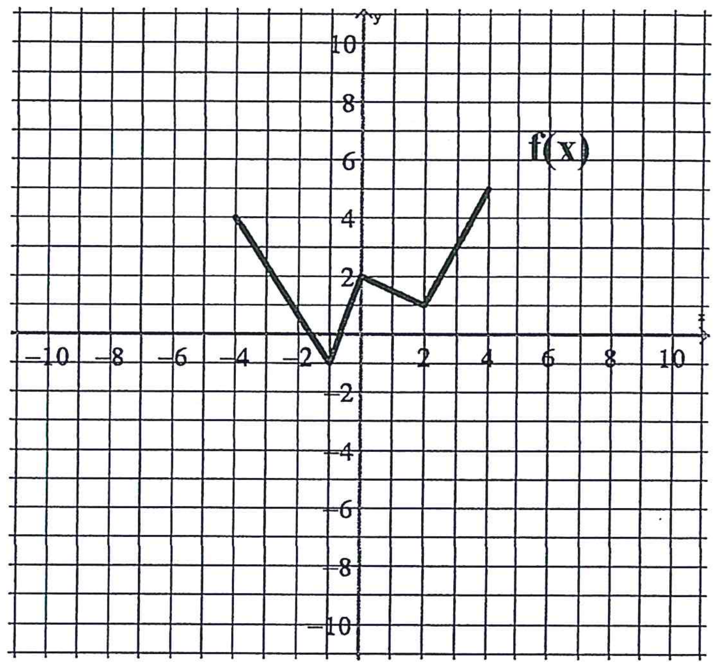
The function f(x) is shown below. In each case it is being transformed to give a new function, g(x). Start by identifying the key points of f(x). Then, using a separate paper, in each case:

a) Build an input/output diagram for g(x) using the equation, and add the key points of f(x)
b) Work outwards in your input output diagram to identify inputs and outputs of g(x)
c) Graph the transformed function g(x) using the points from your input-output diagram
d) Identify the transformations applied to f(x) to result in g(x). Use your graph or I/O diagram.

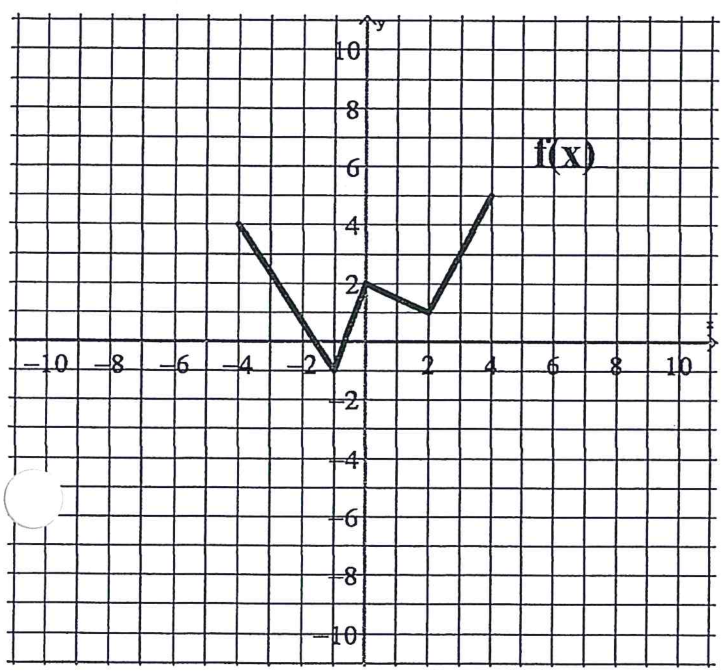
1. $g(x)=2f(x)$



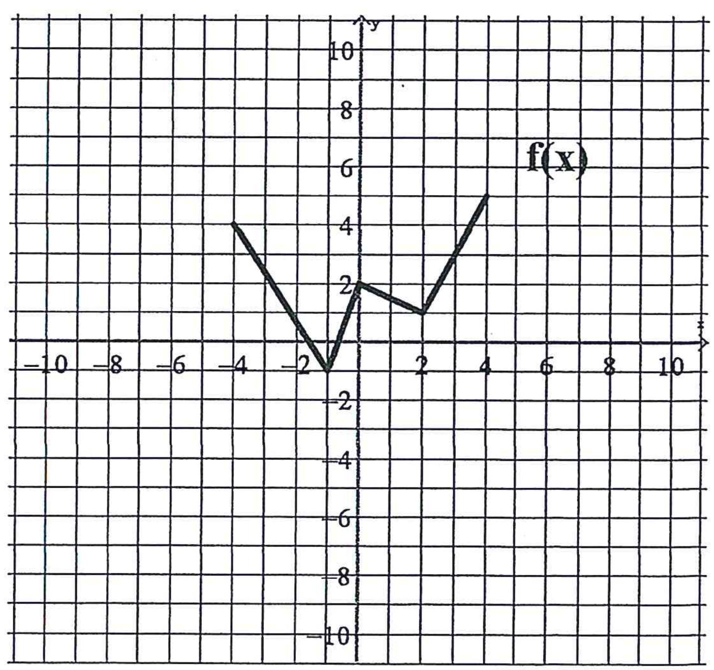
2. $g(x)=f(x-3)$



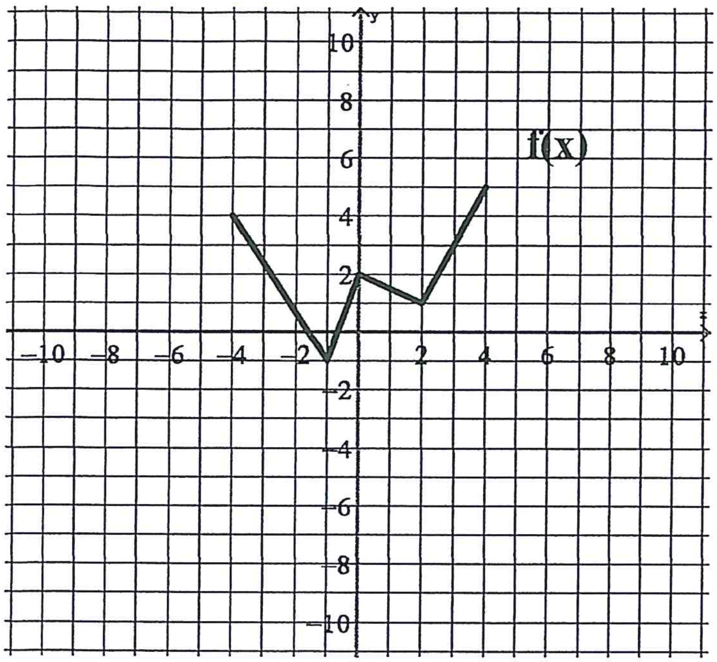
3. $g(x)=f(x)+2$



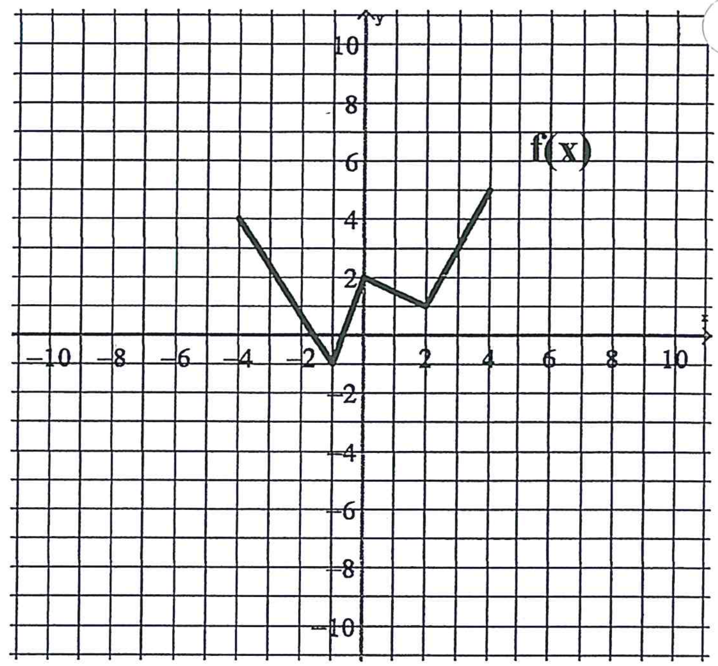
4. $g(x)=f(2x)$



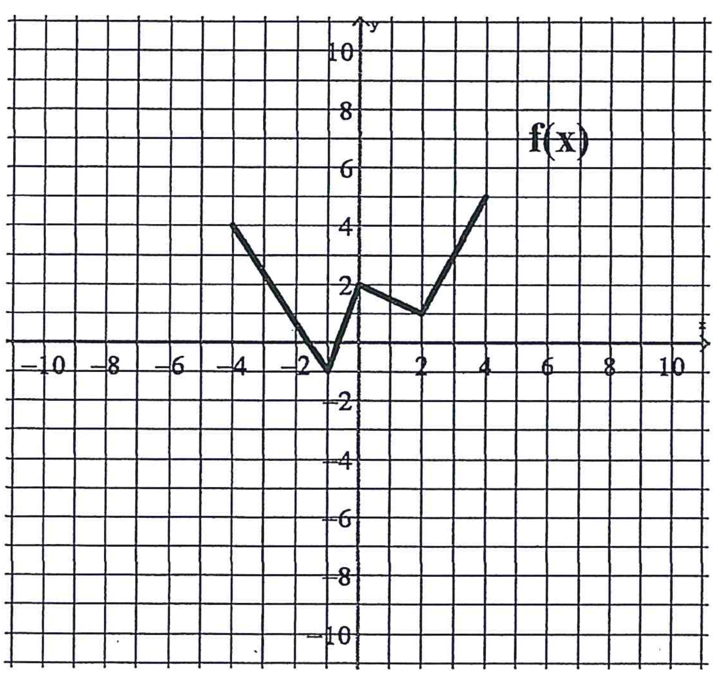
5. $g(x)=2f(x+2)$



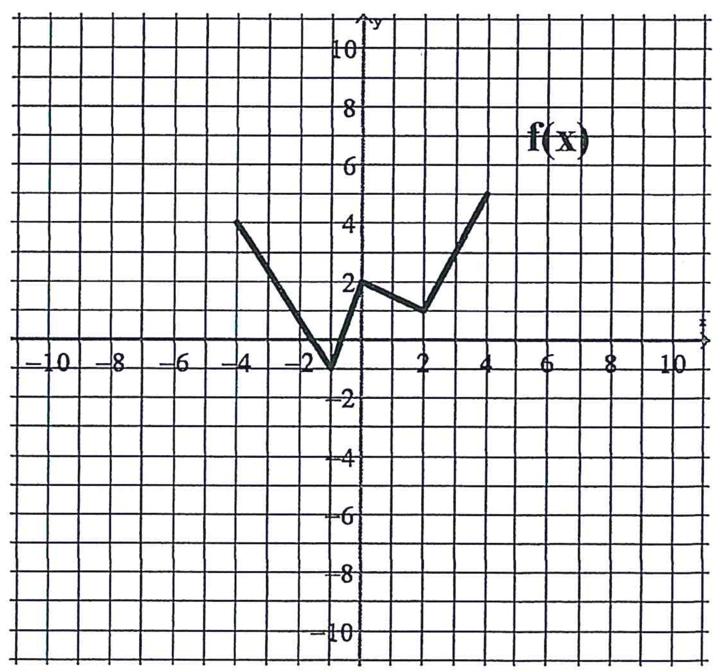
6. $g(x)=\frac{1}{2}f(-x)$



7. $g(x)=f\left(\frac{1}{2}x\right)-2$



8. $g(x)=-2f(0.5(x-1))+1$



Using Input-Output Diagrams to Link Equations and Transformations

Functions convert inputs (x values) into outputs (y values).

x	f(x)
−6	6
1	−2
0	5

"Applying the function f"
(x's become y's)

Functions can convert inputs into outputs using a known mathematical formula. Ex:

$$g(x) = 2x + 3$$

$$h(x) = x^2$$

For the function f, there does not appear to be a logical connection between inputs and outputs.

We can transform the function f to give us a new function h. It will resemble f, but be changed.

$$h(x) = -f(2x + 4) + 1$$

x → [x2] → [+4] → [f] → [x(-1)] → [+1] → $h(x)$

x						
−5	−10	−6		6	−6	−5
−1.5	−3	1		−2	2	3
0	−4	0		5	−5	−4

the x→y boundary

Second they are compressed by 2

First they are moved left 4

First they are reflected over x

Second they are moved up 1

HOW DO <u>THESE</u> INPUTS (x VALUES) COMPARE TO THESE FROM THE PARENT FUNCTION?

HOW DO <u>THESE</u> OUTPUTS COMPARE TO THESE FROM THE PARENT FUNCTION?

Consider the parent function $f(x) = x^2$, as well as the transformed function $g(x) = \frac{1}{2}(x-3)^2 + 5$.

To identify transformations, we will answer the following questions:

1. How are the x-values of the transformed function $g(x)$ different from the x-values of the parent function, $f(x)$?
   → These are vertical transformations
2. How are the y-values of the transformed function $g(x)$ different from the y-values of the parent function, $f(x)$?
   → These are vertical transformations

Here is an input/output diagram for $g(x)$. Given the inputs (x-values) to the function $g(x)$, calculate the associated outputs (y-values)

x → [-3] → [$(\ )^2$] → [$\cdot \frac{1}{2}$] → [+5] → g(x)

x	-3	$(\ )^2$	$\cdot \frac{1}{2}$	+5
1	-2	4	2	7
2	-1			
3				
4				
5				

the x→y boundary

HOW DO THESE INPUTS TO g(x) COMPARE TO THESE INPUTS TO f(x)?

(they are all 3 to the right)

HOW DO THESE OUTPUTS OF g(x) COMPARE TO THESE OUTPUTS OF f(x)

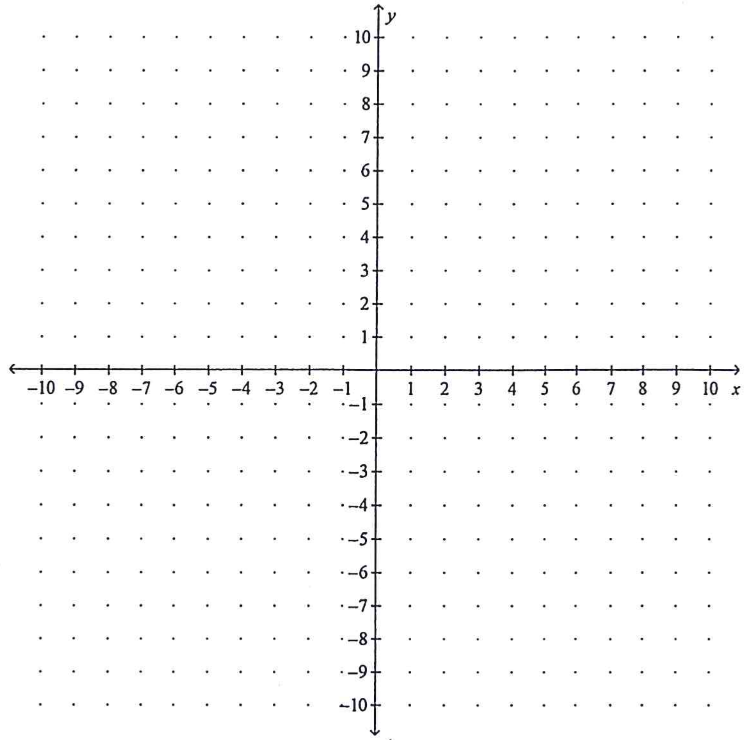
(they are all vertically compressed by 2, then moved up 5)

Input/Output Diagrams, Point Mapping and Transformations

raph each function on the grid to the right, and provide the information requested.

1. $f(x) = x^2$

Input/Output Diagram:



2. $g(x) = (x+6)^2 - 5$

Input/Output Diagram:

Description of transformations to $f(x)$:

Point mapping:

3. $h(x) = -2(x-4)^2 + 7$

Input/Output Diagram:

Description of transformations to $f(x)$:

Point mapping:

(Parent Functions)

Using Function Notation With a Variety of Function Types

Complete the table

Transformed Function in terms of $f(x)$	Transformed Function if Parent Function is: $f(x)=x^2$	$f(x)=\sqrt{x}$	$f(x)=\sin x$
$g(x)=f(x)+3$	$g(x)=x^2+3$	$g(x)=\sqrt{x}+3$	$g(x)=\sin x+3$
$g(x)=f(x)-5$			
$g(x)=f(x-3)$			
	$g(x)=(x+4)^2$		$g(x)=\sin(x+4)$
$g(x)=f(x+2)-1$	$g(x)=(x+2)^2-1$	$g(x)=\sqrt{x+2}-1$	
		$g(x)=\sqrt{x-5}-8$	
$g(x)=2f(x-3)$			
$g(x)=f(-x)$			
$g(x)=-f(x)+3$			$g(x)=-\sin(x)+3$
$g(x)=-f(2x)$			
$g(x)=0.5f(2x)-1$			
		$g(x)=\sqrt{x-4}+3$	
	$g(x)=(-0.5x)^2$		

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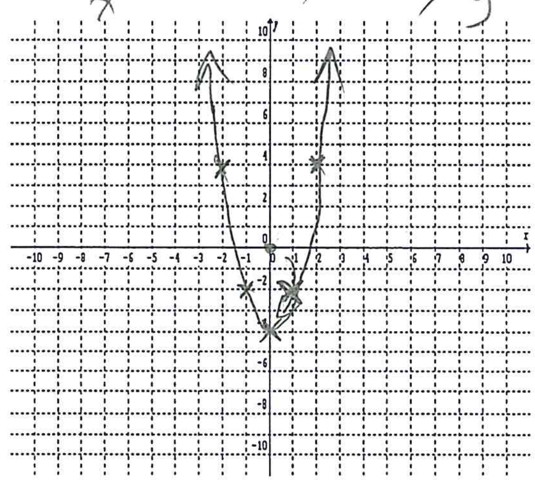
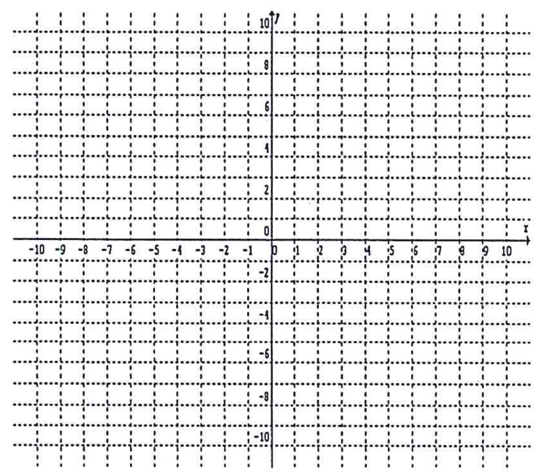
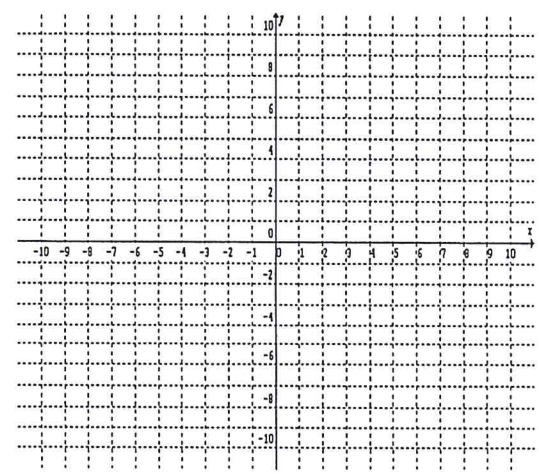
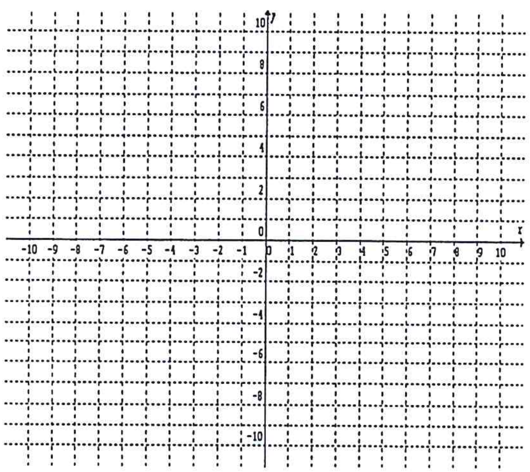
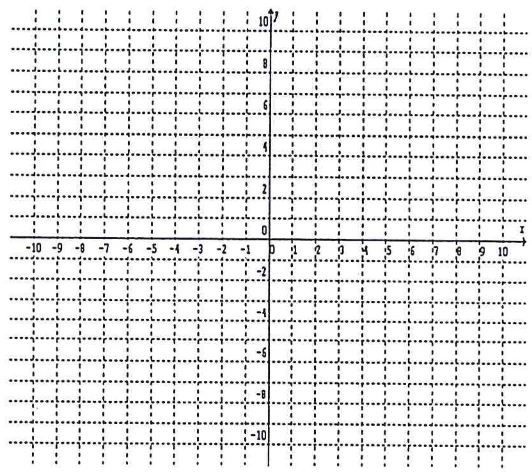
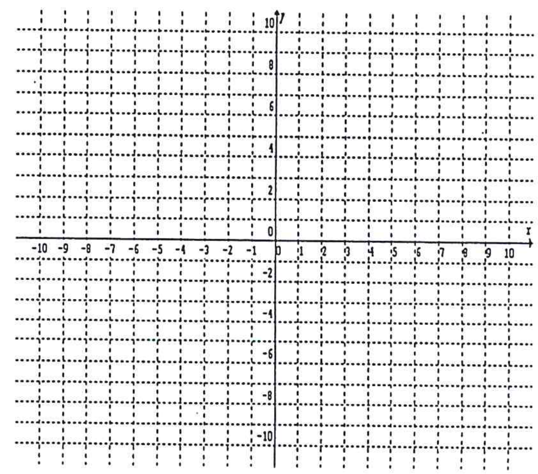
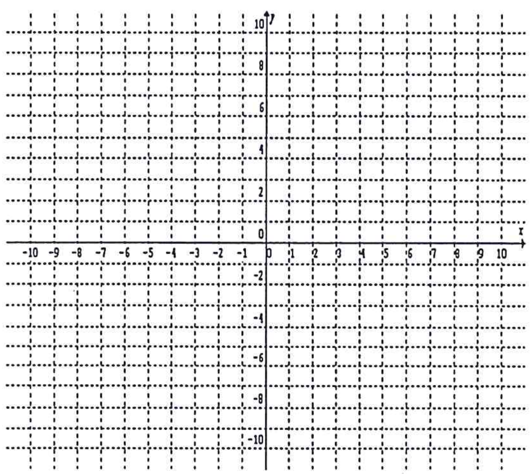
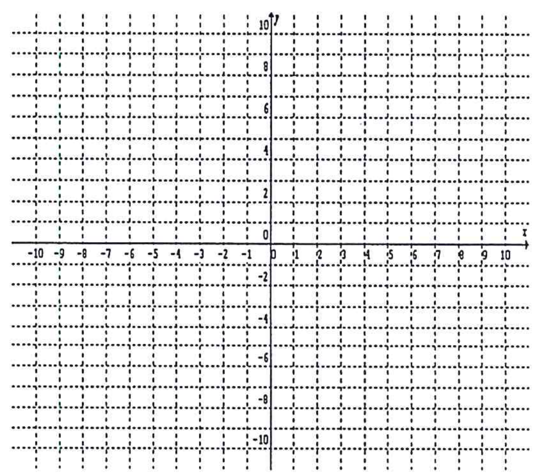
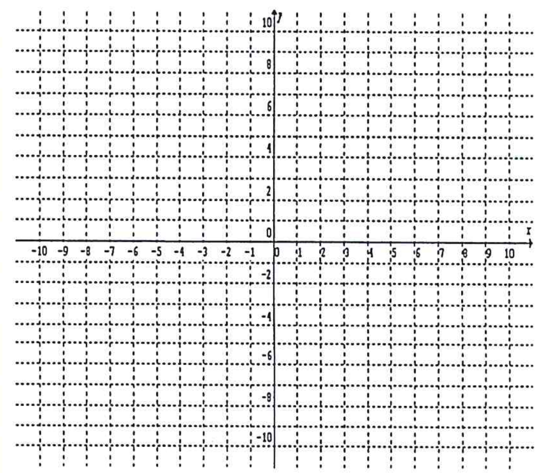
MCR 3U

Different representations of transfo[rm]ations – Quadratic, Root and Piecewise Functions

Parent function	$f(x)=x^2$	$f(x)=x^2$	$f(x)=x^2$
Transformed function g(x)	$g(x)=2x^2-4$	$g(x)=(x+4)^2$	
Transformed function g(x) in terms of f(x)	$g(x)=2f(x)-4$	$g(x)=f(x+4)$	$g(x)=f(2x)$
Description of transformations	– v. stretch by 2 – translation down 4		
Input Output Diagram	$x \to [f] \to [\times 2] \to [-4] \to g(x)$ $x \to [(\ )^2] \to [\times 2] \to [-4] \to g(x)$ x: -2, -1, 0, 1, 2 4, 1, 0, 1, 4 8, 2, 0, 2, 8 y: 4, -2, -4, -2, 4		
Transformation of the point (x, y)			
Graph			

Parent Function	$f(x)=x^2$	$f(x)=x^2$	$f(x)=x^2$
Transformed function *g(x)*		$g(x)=\left(-\frac{1}{2}x\right)^2$	$g(x)=\frac{1}{2}(x+5)^2+1$
Transformed function *g(x)* in terms of *f(x)*	$g(x)=-f(x-3)+5$		
Description of transformations			
Input Output Diagram			
Transformation of the point (x, y)			
Graph			

Parent function	$f(x)=\sqrt{x}$	$f(x)=\sqrt{x}$	$f(x)=\sqrt{x}$
Transformed function *g(x)*		$g(x)=-\sqrt{2x}+3$	
Transformed function *g(x)* in terms of *f(x)*	$g(x)=f(x+4)$		$g(x)=3f(-x)-5$
Description of transformations			
Input Output Diagram			
Transformation of the point (x, y)			
Graph			

Parent function	$f(x)$ in graph below	$f(x)$ in graph below	$f(x)$ in graph below
Transformed function $g(x)$ in terms of $f(x)$	$g(x) = 2f(x) - 5$	$g(x) = -f(x-4)$	$g(x) = f\left[\frac{1}{2}(x-2)\right] + 3$
Description of transformations			
Input Output Diagram			
Transformation of the point (x, y)			
Graph