Changes in Gravitational Energy

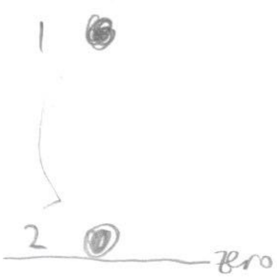
When objects move vertically energy flows in or out of Earth's gravitational field. Let's follow these flows and learn how to model the energy transfers in the system.

A: The Ball Drop and Kinetic Energy

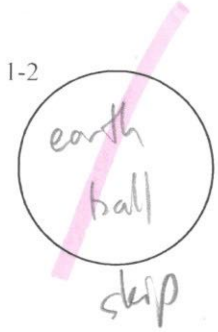
You will drop a basketball through a displacement of your choice (between 0.5 and 1.2 m) and examine the energy changes.

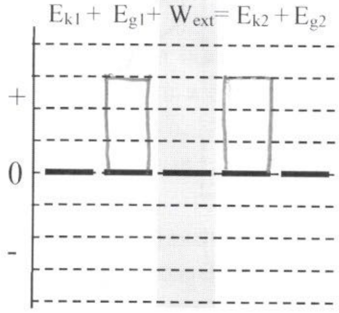
1. **Represent.** Draw a sketch of a ball falling. Event 1 = the ball is released. Event 2 = the ball contacts the ground. Label the two vertical positions y_1 and y_2 (one of these should be the zero-point). Complete the energy-flow diagram and bar chart for the **earth-ball** system.

Sketch



1-2





Work-Energy Equations. Our bar charts help us to think about energy and to construct an equation that relates the energy of a system at two moments in time. The total energy of a system at one moment plus any work equals the total energy of a system at another moment: $E_{T1} + W_{ext} = E_{T2}$. This is called a *work-energy equation* for the system. The bar chart helps us to decide which energies to include in each total. If a particular energy is zero, we don't bother including it.

2. **Represent.** Construct a work-energy equation for the earth-ball system.

$$E_{g1} = E_{k2}$$

3. **Calculate.** Complete your work-energy equation by replacing each energy symbol with its mathematical expression, including event numbers. For example, E_{g1} is replaced with mgy_1.

$$mgy_1 = \frac{1}{2} m v_2^2$$

4. **Calculate.** Use your new work-energy equation to find the speed of the ball when it contacts the ground. Remember to algebraically isolate for v_2 first! Something neat will happen!

$$\cancel{m} g y_1 = \frac{1}{2} \cancel{m} v_2^2$$

$$g y_1 = \frac{1}{2} v_2^2$$

$$2 g y_1 = v_2^2$$

$$v_2 = \sqrt{2 g y_1}$$

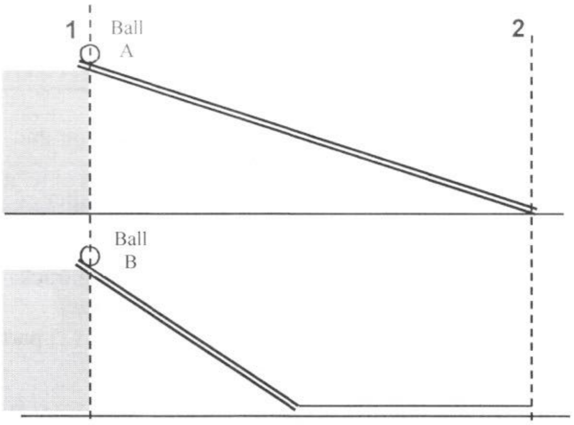
5. **Test.** Use the motion detector to measure the speed of the ball when it contacts the ground.

6. **Evaluate.** How does your measured value for the speed compare with your prediction? What might be responsible for a small difference?

Air resistance, hand is moving, measuring device might not be accurate.

B: The Ramp Race

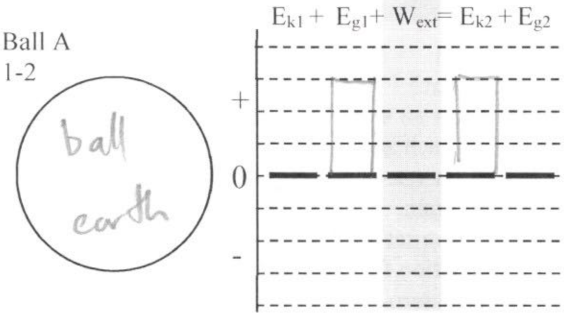
Your teacher has two tracks set up at the front of the class. One track has a steep incline and the other a more gradual incline. Both start at the same height and end at the same height. Friction is very small and can be neglected. There are two important events: (1) Ball A and B are released, (2) Each ball reaches the end of the track.



1. **Reason.** What energy changes take place as the ball travels down the incline?

Grav. potential → kinetic

Ball A
1-2

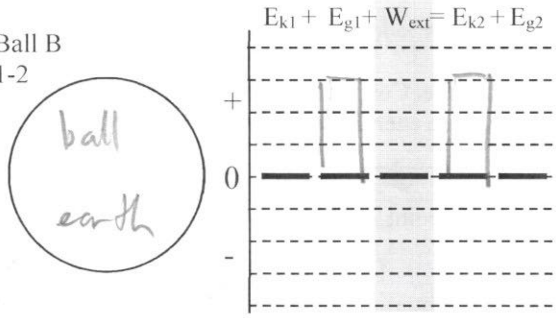


2. **Represent.** Complete an energy flow diagram and an energy bar chart for each ball for the interval 1-2. **System = Ball, Earth**

3. **Explain.** Describe and explain any similarities between the two sets of diagrams.

Exactly the same

Ball B
1-2



4. **Predict.** Use your energy bar chart to predict which ball will have the greater speed at moment 2. Explain.

Same speed – all energy converted to kinetic in both cases

5. **Observe.** (*as a class*) Record your observations when: (a) the two balls are released at the same time

and (b) when they reach the end of the track at the same time.

6. **Reason.** Albert says, "I don't understand why ball B wins the race. They both end up traveling roughly the same distance and ball A even accelerates for more time! It should be faster!" Based on your observations and understanding of energy, help Albert understand.

Ball B had all its potential energy converted to kinetic energy sooner, so it reached max speed earlier.

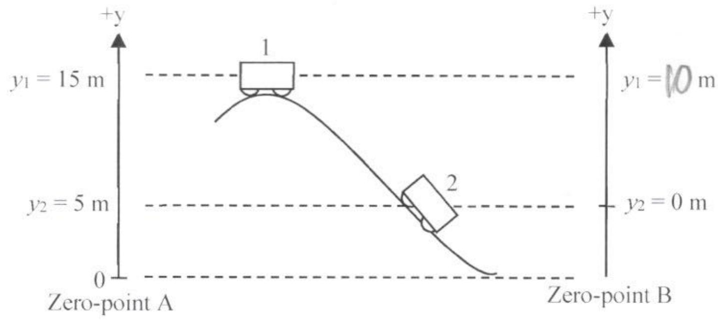
Path Independence. The amount of energy the flows in or out of the gravitational field **does not depend on the path** taken by the object. It only depends on the object's change in vertical position (displacement). The property is called *path independence* – any path between the same vertical positions will give the same results. This happened because gravity does no work on an object during the horizontal parts of the object's motion.

optional

Homework: Changes in Gravitational Energy   Name:

The value for the gravitational energy depends on the choice of the zero-point. If two people choose a different zero-points, will their calculations predict different things? Let's see!

A 100 kg rollercoaster cart rolls down a curving track. It starts from rest at the top. We will examine two moments in time: (1) at the top of the track and (2) part way down. **System = cart, Earth**

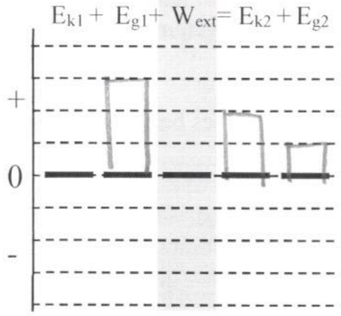


1. **Calculate.** Find the value of y_1 using zero-point B.

   10 m

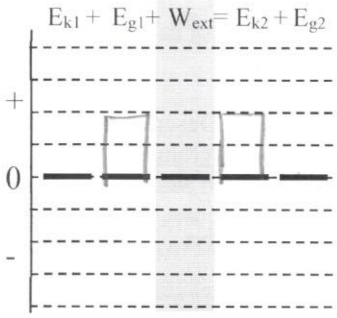
2. **Represent.**
   (a) Draw an energy bar chart for each zero-point.
   (b) Draw one energy flow diagram.
   (c) Construct a work-energy equation for the system for each zero-point.

1-2



Equation:

$E_{g1} = E_{k2} + E_{g2}$



Equation:

$E_{g1} = E_{k2}$

3. **Calculate.** Complete the chart below. Calculate the gravitational energies of the system according to each zero-point. Use these energies to determine how much kinetic energy and speed the cart has a moment 2.

space

	E_{g1}	E_{g2}	E_{k2}	v_2
Zero-point A	$mgh = (100)(10)(15) = 15000$ J	$(100)(10)(5) = 5000$ J	$\frac{1}{2}mv^2 = \frac{1}{2}(100)v^2 = 50v^2$	$15000 = 5000 + 50v^2$; $10000 = 50v^2$; $200 = v^2$; $v = 14.1$ m/s
Zero-point B	$mgh = (100)(10)(10) = 10000$ J	0	$\frac{1}{2}mv^2 = \frac{1}{2}(100)v^2 = 50v^2$	$E_{g1} = E_{k2}$; $10000\text{ J} = 50v^2$; ⋮; $v = 14.1$ m/s

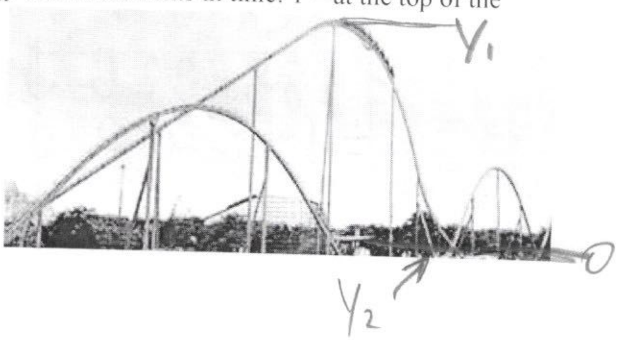
4. **Explain.** Use both the calculations and the bar charts to explain why the choice of zero-point did not affect the results of the calculation.

Changes in Gravitational Energy. Only *changes* in gravitational energy affect predictions using energy techniques. That is why we can set any vertical position as the zero-point. The vertical displacement of the object does not depend on the choice of origin and therefore the *change* in gravitational potential energy does not depend on it either!

The Conservation of Energy

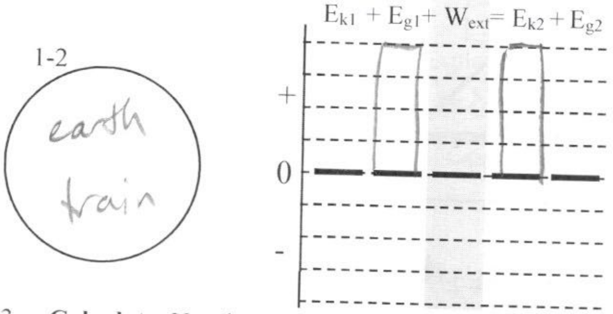
A: The Behemoth

A recent rollercoaster at Canada's Wonderland is called "The Behemoth" due to its 70.1 m tall starting hill. Assume the train is at rest when it reaches the top of the first hill. We will compare the energy at two moments in time: 1 = at the top of the first hill and 2 = at ground level after the first hill.



1. **Represent.** Choose a zero-point for gravitational energy. Label on the photo the vertical positions y_1 and y_2.

2. **Represent.** Draw an energy bar chart and flow diagram for the earth-train system. Write down a complete work-energy equation that relates the energies of the system at moment 1 with moment 2. Only write down the energy terms that are not zero.



Work-Energy Equation

$E_{g1} = E_{k2}$

space, isolate.

3. **Calculate.** Use the energy equation to find the speed of the rollercoaster at moment 2 in km/h. (Remember to isolate for the unknown first.)

$E_{g1} = E_{k2}$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$(10)(70.1) = \frac{1}{2}v^2$

$701 = \frac{1}{2}v^2$

$2(701) = v^2$

$v^2 = 1402$

$\vec{v} = 37.4$ m/s $\xrightarrow{\times 3.6}$ $\vec{v} = 134.8$ km/h $\doteq 135$ km/h

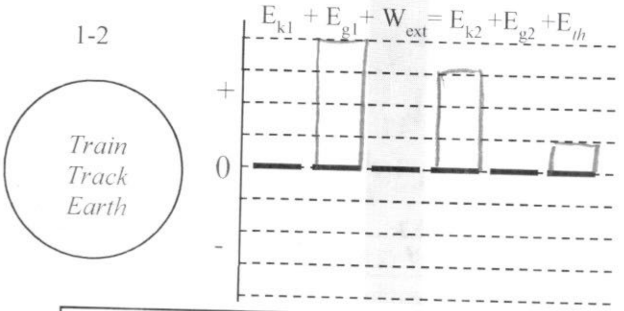
4. **Reason.** The official statistics from the ride's website give the speed after the first drop as 125 km/h. Is our model giving a reliable result? What assumption in our model might need to be changed to get a better prediction?

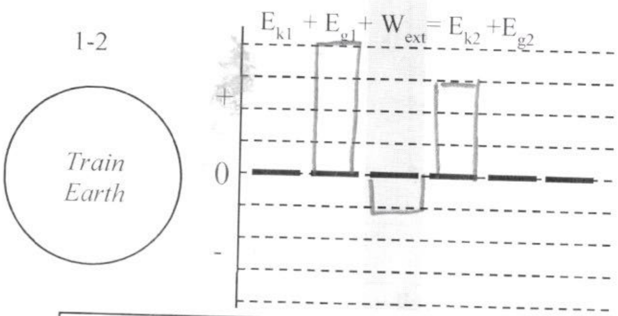
Our model could include energy loss due to friction, air, shaking, etc.

Thermal Energy. When two objects slide against another, energy is transferred into *thermal energy* (E_{th}) due to a friction interaction. The two sliding objects will warm up, which means the thermal energy is shared between them. You may either treat thermal energy as internal $E_{th} = F_f\,\Delta d$ for the *earth-train-track system* or external $W_{ff} = F_f\,\Delta d$ for the *train-track* system.

could skip.

5. **Represent.** Draw a new energy storage bar graph and an energy flow diagram that takes into account the effects of friction for both the *earth-train-track* system and the *earth-train* system. Write down the new work-energy equations.





Work-Energy Equation

$E_{g1} = E_{k2} + E_{th}$

Work-Energy Equation

$E_{g1} - W_{ext} = E_{k2}$

→ 2 sig digits

6. **Calculate.** Use the train mass, $m_t = 2.7 \times 10^3$ kg to determine the amount of thermal energy at moment 2. Hint: which velocity value should you use for E_{k2}?

In theory
$E_k = \frac{1}{2}mv^2$
$= \frac{1}{2}(2700)(37.4)^2$
$= 1888326$ J
$\doteq 1900000$ J

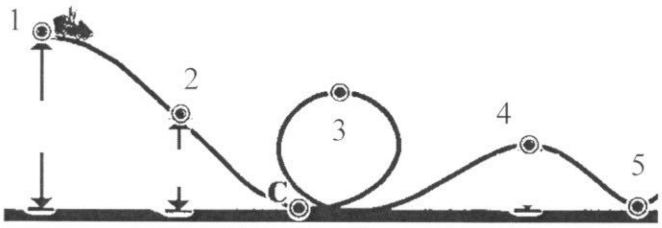
In Reality $v = 125$ km/h $= 34.7$ m/s
$E_k = \frac{1}{2}(2700)(34.7)^2$
$= 1627604$ J
$\doteq 1600000$ J

Answer: v_2 = 14 m/s??

$1900000 - 1600000 = 300000$ J

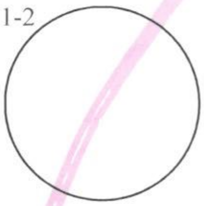
B: The Glebe Flyer

Rumour has it that a rollercoaster is going to be built in our school's courtyard. Plans leaked to the media show a likely design. The train starts from rest at moment 1 located 45 m above the ground. At moment 2 it is located 10 m above the ground. For all our calculations, we will assume that the force of friction is negligible.



1. **Solve.** Label the important vertical positions on the diagram. Complete the diagram and chart. Determine the rollercoaster's speed at moment 2.

1-2



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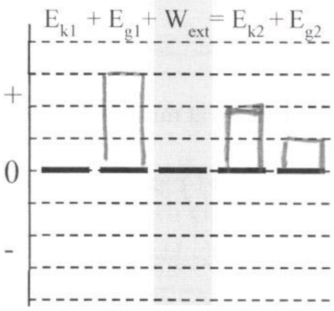
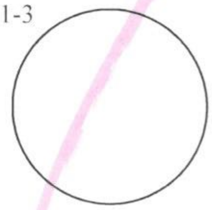


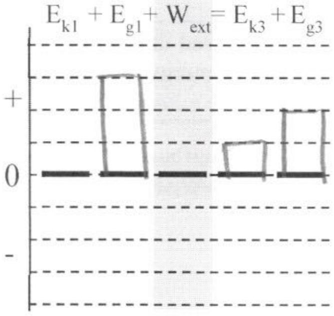
Equation
$E_{g1} = E_{k2} + E_{g2}$

$\cancel{m}gh_1 = \cancel{m}v_2^2 + \cancel{m}gh_2$
$(10)(45) = v_2^2 + (10)(10)$
$450 = v^2 + 100$
$350 = v_2^2$
$v_2 = 18.7$ m/s → $V_2 \approx 19$ m/s

2. **Solve.** Label the important vertical positions on the diagram. Moment 3 is the top of the loop-de-loop and is located 17 m above the ground. Complete the diagram and chart. Determine the rollercoaster's speed at moment 3.

1-3





Equation
$E_{g1} = E_{k3} + E_{g3}$

Over the loopdeloop its velocity is 17 m/s.
How tall is the loop deloop?

$\cancel{m}gh_1 = \cancel{m}gh_2 + \frac{1}{2}\cancel{m}v_2^2$
$(10)(45) = (10)h_2 + \frac{1}{2}(17)^2$
$450 = 10h_2 + 144.5$
$305.5 = 10h_2$ → $h_2 = 30.55$ m
$h_2 \doteq 31$ m

Path Independence. The full loop-de-loop motion involves some interesting but challenging physics. But by using energy techniques, we did not have to consider those complications at all. When there are no transfers to thermal energy, we don't have to worry about the path an object takes during the interval, no matter how complex. Wow!

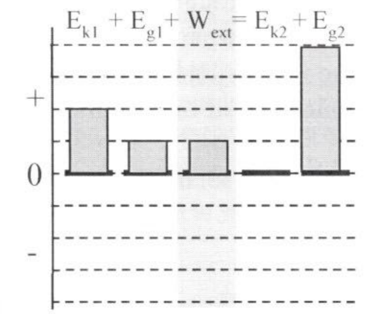
remove

Homework: The Conservation of Energy Name:

1. **Reason.** A block is attached to a rope so you can raise or lower it vertically. An energy bar chart illustrates the energies at two moments in time while it is being raised or lowered.
   (a) Use the bar chart to explain what is happening to the speed and position of the block.
   (b) Draw an energy flow diagram and write a complete work-energy equation for each interval.

yes

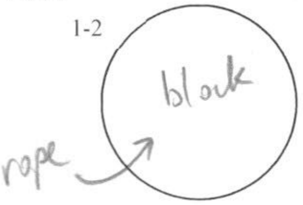
leave these?



Explain:

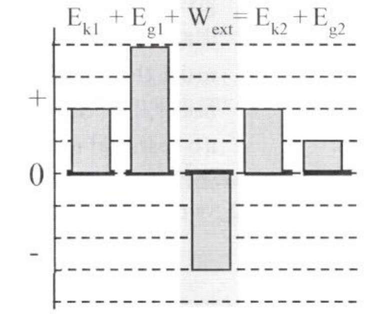
being raised, starts with velocity but slows down.

Flow:



Work-Energy Equation:

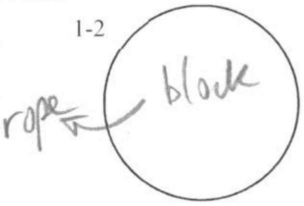
$E_{k1} + E_{g1} + W_{ext} = E_{g2}$



Explain:

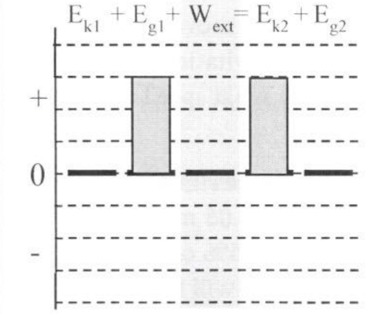
being lowered at constant velocity

Flow:



Work-Energy Equation:

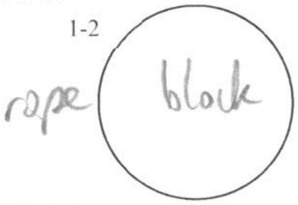
$E_{k1} + E_{g1} - W_{ext} + E_{k2} + E_{g2}$



Explain:

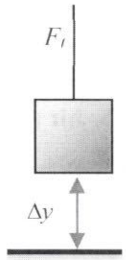
rope is letting block fall, applying no force

Flow:



Work-Energy Equation:

$E_{g1} = E_{k2}$

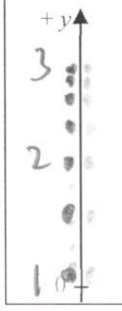


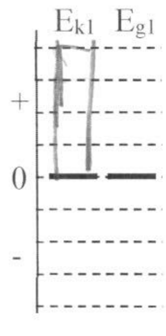
$= 0.20\,kg$

2. **Represent and Calculate.** You throw a 200 g ball upwards. It leaves your hand with a speed of 10 m/s. We choose a vertical origin at the vertical position where the ball is released from your hand. We examine three moments in time: (1) it leaves your hand, (2) it is halfway up, and (3) it is at its highest point.
   (a) Draw a motion diagram and label these moments.
   (b) For each moment in time, complete an energy bar chart for the earth-ball system.
   (c) Calculate the energies at each moment and find the total energy of the system. Show your work.

space

Motion Diagram

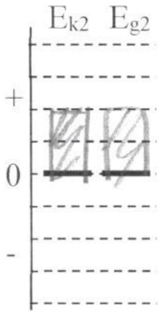




$E_{k1} = \frac{1}{2}(0.2)(10)^2$
$= 10\,J$

$E_{g1} = 0$

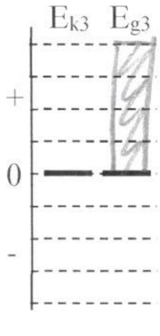
$E_{T1} = 10\,J$



$E_{k2} = 5\,J$

$E_{g2} = 5\,J$

$E_{T2} = 10\,J$



$E_{k3} = 0\,J$

$E_{g3} = 10\,J$

$E_{T3} = 10\,J$

skip

(d) How does the total energy compare at each moment in time? Does this make sense?