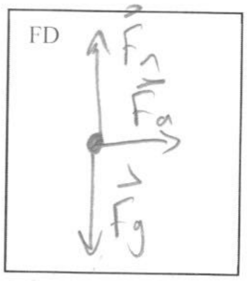
Doing Work!

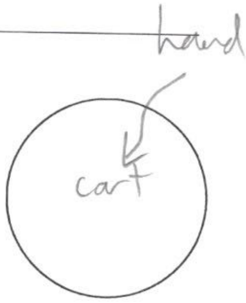
How do we track the transfer energy into or out of a system?

A: The Energetic Cart

We will be using a cart in this investigation, and we will assume the force of friction between the cart and table is zero.

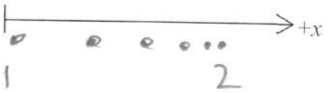
1. **Describe and Represent.** We want some energy to flow from your hand to the cart. Describe how you can do this. Draw a motion diagram, a force diagram, and an energy flow diagram during this process (while it gains kinetic energy). **System = cart**

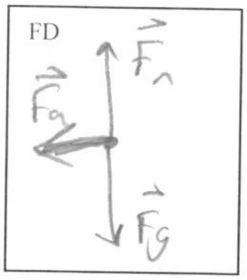


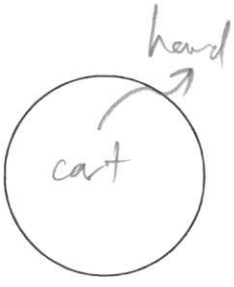


We apply a force (ie push)

2. **Describe and Represent.** The cart is initially moving quickly and we want energy to flow from the cart to your hand. Describe how you can do this. Draw an MD, FD, and an energy flow diagram during this process. **System = cart**







We apply a force in the opposite direction it is moving in

3. **Demonstrate**. Use the cart and show these two situations to your teacher. Move on to the next questions while you wait.

Mechanical Work. Energy can be transferred into or out of a system by an external force. We say that the external force does work on the system. If the system loses energy, we say the force does negative work. If the system gains energy, we say the force does positive work. Energy is a scalar quantity; positive or negative work does not indicate a direction, it only indicates a gain or loss of energy for the system.

4. **Explain.** In which case above was the work positive or negative. Explain.

#1 - system (cart) gained (kinetic) energy

#2 : cart lost kinetic energy.

5. **Reason.** Think about the first situation above. What do you think would happen to the amount of energy that flows into the system if your hand exerted the same force for twice the distance? Explain.

It would increase, as we are adding energy for a longer amount of time.

6. **Reason.** Think about the second situation above. Suppose the cart was moving at the same speed, but now the force you exerted was twice as large. What would happen to the stopping distance? Explain.

Would stop sooner - the stronger ~~to~~ push back

The Definition of Work. The work done by a force on a system (W) depends on three quantities: the size of the force (F), the displacement of the system (Δd) and the angle between the force vector and the displacement vector (θ). These are related by the expression, $W = |F||\Delta d|\cos\theta$. The units of work may be expressed as N·m, but this is equivalent to the unit joules (J) for energy.

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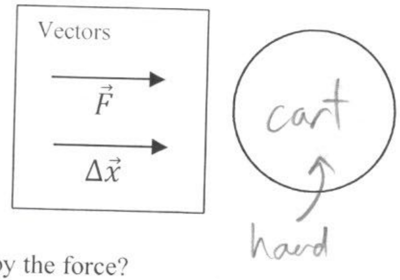
B: Working the Angles

1. **Reason and Calculate.** A cart with a mass of 0.70 kg is initially at rest. Then it is pushed horizontally by a hand with a force of 10 N. Friction is negligible.
   (a) Vector arrows showing the direction of the force from the hand and the displacement of the cart are drawn for you. Draw an energy flow diagram for the system of the cart.



   (b) What is the angle between the two vectors? (The angle represents the *difference* in direction of the two vectors)

   $0°$

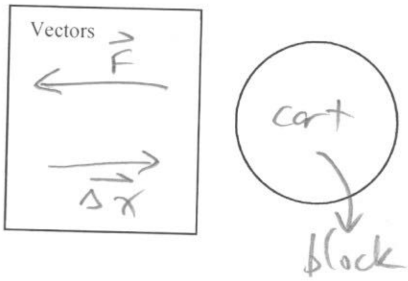
   (c) After it moves a distance of 0.40 m, how much work (in joules) has been done by the force?

   $W = Fd\cos\theta$
   $= (10)(0.4)\cos 0° = 4\text{ J}$

   (d) Interpret the sign of the value for the work that you calculated. How much kinetic energy do you think the cart has now?

   System gained energy. Cart now has 4 J of kinetic energy

2. **Reason and Calculate.** The same cart is rolling along a table and is released. It collides with a block that exerts a 12 N stopping force on the cart. It rolls 0.35 m while stopping.
   (a) Draw vector arrows for the block's force and the displacement of the cart. Draw an energy flow diagram for the system of the cart.



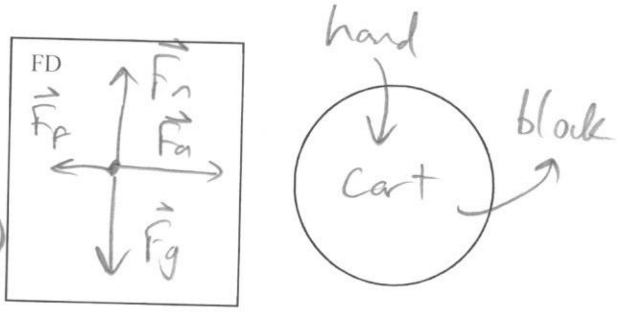
   (b) What is the angle between the two vectors? What is the work done by the block's force while bringing the cart to rest?

   $180°$ $W = Fd\cos\theta$
   $= (12)(0.35)(\cos 180°) = -4.2\text{ J}$

   (c) Interpret the sign of the value for the work that you calculated. How much kinetic energy do you think the cart originally had?

   Energy removed from system.
   Cart had 4.2 J

3. **Reason and Calculate.** Now you push on the cart for 0.50 m while the cart pushes against the block. The block's force is still 12 N and you push horizontally with a force of 15 N.
   (a) Draw energy flow and force diagrams for the system of the cart.



   (b) Calculate the work done by each force acting on the system.

   Hand: $W = (15)(0.5) = 7.5\text{ J}$ Block: $W = (12)(0.5)(\cos 180°) = -6\text{ J}$

   (c) What is the total work done on the system? How much kinetic energy did the system gain during this process?

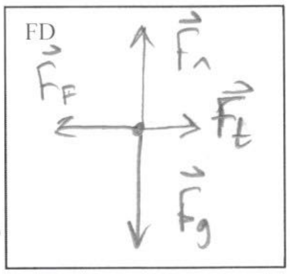
   $7.5\text{ J} + (-6\text{ J}) = 1.5\text{ J}$ energy gained

Net Work – Kinetic Energy Theorem. The *net work* is the sum of all the work done on the particle. If the net work is positive, the particle *gains* kinetic energy. If the net work is negative, the particle *loses* kinetic energy. This idea is called the *net work - kinetic energy theorem* and is represented by the expression: $W_{net} = E_{k2} - E_{k1} = \Delta E_k$. Note that this is the same as finding the work done on the particle by the net force vector: $W_{net} = |F_{net}||\Delta d|\cos\theta$.

Homework: Doing Work! Name:

You are pulling a wagon. It is a bit tiring, but everything is going well until you try to pull it through a patch of mud. The wagon slows and stops. We will focus on the interval while the wagon is slowing in the mud. We will model the effect of the mud on the wagon like a friction force.

1. **Represent**. Draw an energy flow diagram and a force diagram for the system of the wagon.



2. **Describe.** Which forces cause energy to flow in or out of the system?

Friction - energy out
Applied - energy in (tension from rope)

3. **Reason.** Use the new expression for work ($W = |F||\Delta d|\cos\theta$) to help complete the chart for each force acting on the car.

Force	θ	Sign of work? (+, - or 0)	Flow of energy? (in, out or none)
F_t	0°	+	in
F_f	180°	−	out
F_n	90°	0	none
F_g	90°	0	none

4. **Reason.** Overall, is the system gaining or losing energy? Explain how you know.

Losing energy...cart was moving at start, then it was stopped.

5. **Reason.** Do forces acting perpendicular to the displacement of an object transfer energy in or out of a system? Explain.

Neither - normal & gravity force did not affect energy at all (since $\cos 90° = 0$ and $\cos 270° = 0$)

6. **Reason.** Did the sign of the work depend on our choice of our positive/negative direction choice? Explain.

Yes - when force & displacement were in same direction, work was positive. (and vice versa)

7. **Calculate.** The size of your pulling force was 59 N. The friction force from the mud was 75 N. The wagon initially had 8.3 J of kinetic energy. How far did it travel through the mud before stopping?

$W = |F||d|\cos\theta$

$-8.3 = (16)(d)\cos 180°$

$d = \frac{-8.3}{16} = 0.52 \text{ m}$

$F_{net} = 59 + (-75)$

$F_{net} = -16 \text{ N}$

$|F_{net}| = 16 \text{ N}$

8. Now you pull the wagon really hard and it starts to speed up.
   (a) **Reason.** Is this situation accurately described by the FD and energy flow diagram above? Explain what changes you would need to make.

   length of tension force vector would need to be increased to be longer than friction vector.

   (b) **Reason.** Is the wagon gaining or losing kinetic energy? Use the *kinetic energy-net work theorem* to determine the sign of the net work.

   Gaining, so sign of net work would be +.

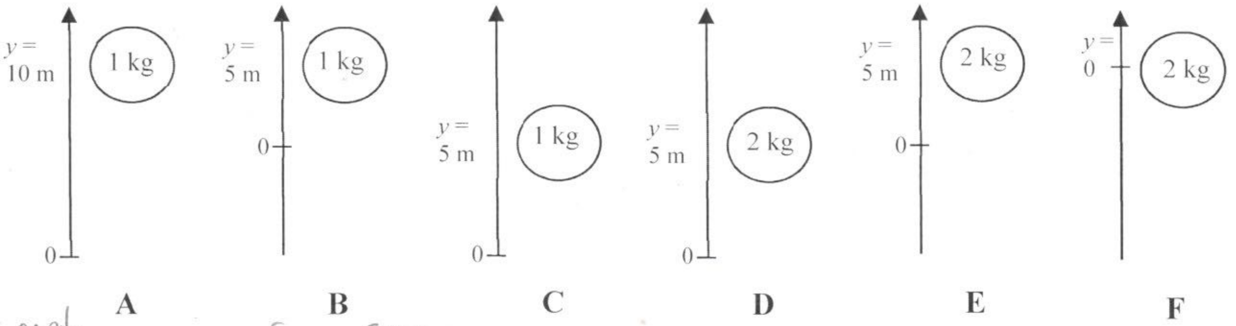
   (c) **Reason.** How does the amount of energy transferred by each force compare in this situation?

   More energy transferred in due to pulling than out due to mud.

Homework: Measuring Energy Name:

A: Comparing Gravitational Energies Using $E_g = mgy$

1. **Reason.** Six objects and their vertical positions relative to a zero-point are shown. Rank the gravitational energies of each object. Explain your ranking.



$E_g = mgh = (1)(10)(10) = 100$ J

$E_g = 50$ J

$E_g = 50$ J.

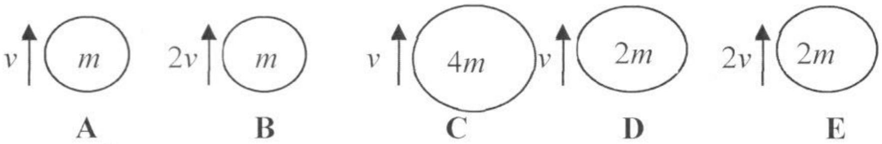
$E_g = (2)(10)(5) = 100$ J

$E_g = 100$ J

$E_g = 0$ J

B: Comparing Kinetic Energies Using $E_k = \frac{1}{2}mv^2$

1. **Reason.** The velocity and mass of five objects is shown to the right. Suppose $v = 1$ m/s and $m = 1$ kg. Rank the amount of kinetic energy each object has. Explain your ranking.



$E_K = \frac{1}{2}(1)(1)^2 = \frac{1}{2}$ J

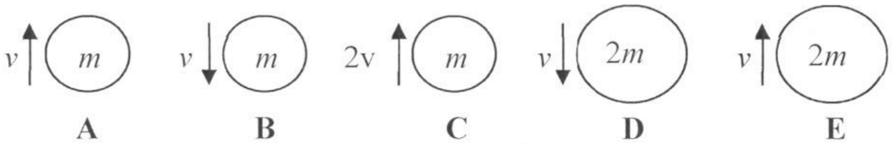
$\frac{1}{2}(1)(2)^2 = 2$ J

$\frac{1}{2}(4)(1)^2 = 2$ J

$\frac{1}{2}(2)(1)^2 = 1$ J

$\frac{1}{2}(2)(2)^2 = 4$ J

2. **Reason.** The velocity and mass of five objects is shown to the right. Suppose $v = 1$ m/s and $m = 1$ kg. Up is positive and down is negative. Rank the amount of kinetic energy each object has. Explain your ranking.



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↳ doesn't depend on direction (up/down)

C: Calculating Energies

1. **Reason**. A friend shows you the results of his calculations. (a) Explain what errors he made and (b) correct his work.

$m = 250$ g
$v_1 = 5.0$ km/h
$y_1 = 3.4$ m

$E_{g1} = mgy_1 = (250\text{ g})(9.8\text{ N/kg})(3.4\text{ m}) = 8330$ J

convert to kg

$E_g = (0.25)(9.8)(3.4) = 8.33$ J

$5\text{ km/h} = 1.39$ m/s

$5 \frac{km}{h}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$

$E_{k1} = \frac{1}{2}mv_1^2 = (0.5)(250\text{ g})(5.0\text{ km/h})^2 = 3125$ J

→ kg ↳ convert to m/s

$E_K = (0.5)(0.25)(1.39)^2 = 0.24$ J

2. **Reason.** Two identical test cars are driving down a test track and hit their brakes at the same position. One car is travelling at twice the speed as the other. (a) Compare the kinetic energies of the two cars. (b) Use the idea of work to explain how much farther the faster car travels while braking.

– Faster car has 4x the kinetic energy because velocity is squared.

W = Fd

If it has 4 the energy, it will 4x the distance to brake.

Answer: C#1 E_{g1} = 8.33 J and E_{k1} 0.241 J

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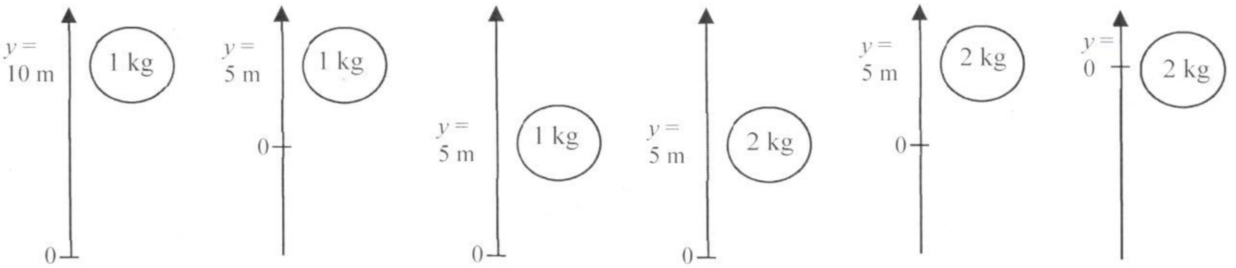

A	B	C	D	E	F
$E_g = mgh = (1)(10)(10) = 100$ J	$E_g = 50$ J	$E_g = 50$ J.	$E_g = (2)(10)(5) = 100$ J	$E_g = 100$ J	$E_g = 0$ J

B: Comparing Kinetic Energies Using $E_k = ½mv^2$

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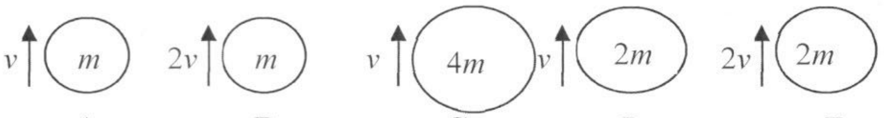

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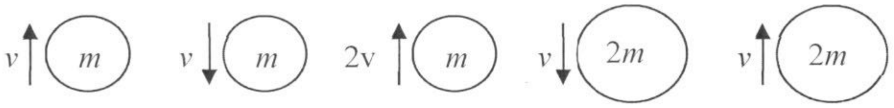

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