SPH3U: Force Problem Solving

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$
$$\Delta\vec{x} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}(\Delta t)^2$$
$$\Delta\vec{x} = \vec{v}_2\Delta t - \tfrac{1}{2}\vec{a}(\Delta t)^2$$
$$\Delta\vec{x} = \tfrac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$$
$$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{x}$$

Forces help us to understand why things move the way they do. Newton's 2nd Law, $F_{net} = ma$, is the law of cause and effect: it relates the causes of motion (forces) with the effects (acceleration). To solve problems involving forces and motion you will use Newton's 2nd Law, $F_{net} = ma$ and the BIG 5 equations.

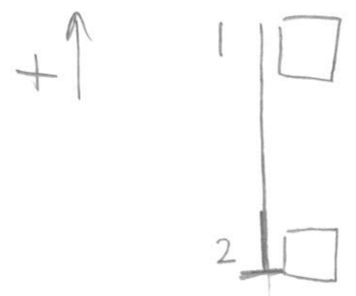
A: The Elevator

An elevator and its load have a combined mass of 1600 kg. It is initially moving downwards at 3.2 m/s. When the elevator passes the second floor, a motor attached to the cable supporting the elevator causes it to slow down through a distance of 8.7 m, allowing the people to get out on the first floor. Complete the parts of our solution process below.

question? what is tension in cable?

A: Pictorial Representation

Sketch, coordinate system, label givens with symbols, conversions, describe events



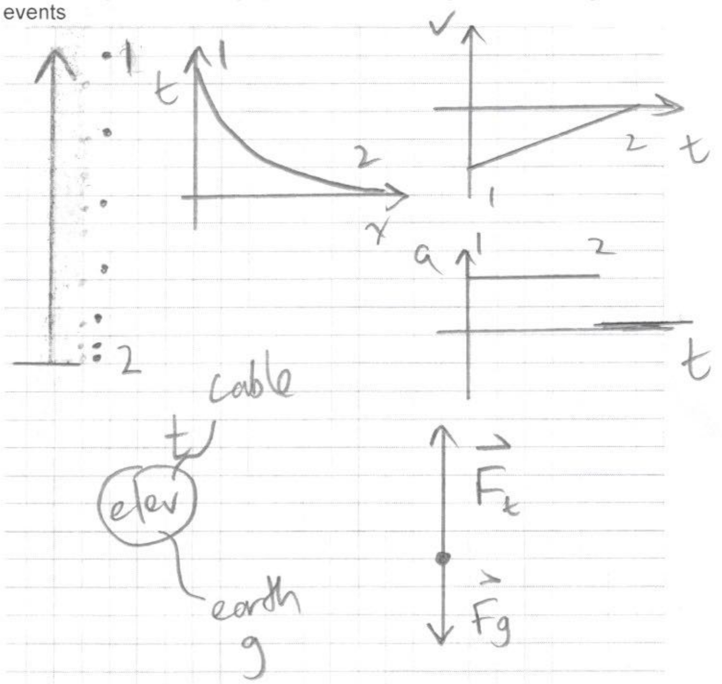
$\vec{v}_1 = -3.2$ m/s
$m = 1600$ kg
$\vec{x}_1 = 8.7$ m
$\vec{a} = ?$
$\vec{v}_2 = 0$ m/s
$\vec{x}_2 = 0$ m

we will use first floor as origin

event 1: passes 2nd
event 2: arrives at first

C: Physics Representation

Motion diagram, motion graphs, interaction diagram, force diagram, events



1. **Describe.** While it is slowing down, what is the elevator interacting with?

Gravity pulling down, cable pulling up.

B: Word Representation

Describe motion (no numbers), explain why, assumptions

> Did you explain *why* it slows down?
> What are we assuming about the acceleration?

- assume constant upwards acceleration
- assume no friction or air resistance
- velocity is downwards, acceleration is in opposite direction (upwards), so slows down

D: Mathematical Representation

Number and describe steps, complete equations, substitutions with units, final statement with units, direction and significant digits

Is the size of the tension force reasonable? How can you tell compared with gravity?

1, Find acceleration

$\vec{v}_1 = -3.2\ m/s$

$\vec{v}_2 = 0\ m/s$

$\Delta x_{12} = -8.7\ m$

$\vec{a} = ?$

~~$\Delta t =$~~

$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{x}$

$0^2 = (-3.2\ m/s)^2 + 2\vec{a}(-8.7\ m)$

$0 = 10.24\ \frac{m^2}{s^2} + (-17.4\vec{a})$

$(17.4\ m)(\vec{a}\ m/s^2) = 10.24\ \frac{m^2}{s^2}$

$\vec{a} = \frac{10.24}{17.4}\ m/s^2$

$\vec{a} = 0.5885\ m/s^2$

$\boxed{\vec{a} = 0.59\ m/s^2\ [up]}$

2. Find $\vec{F}_{net\,y}$

$\vec{F}_{net\,y} = m\vec{a}$

$\vec{F}_{net\,y} = (1600\ kg)(0.59\ m/s^2)$

$\boxed{\vec{F}_{net\,y} = 944\ N}$

3. Find $\vec{F}_t$

$\vec{F}_g = m\vec{g}$

$\vec{F}_{net\,y} = \vec{F}_g + \vec{F}_t$

$944\ N = (1600)(-10) + F_t$

$944\ N = -1600\ N + F_t$

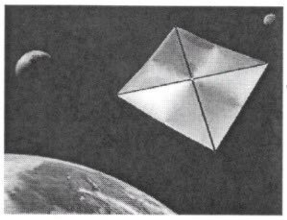
$\boxed{F_t = 2544\ N}$

space

$\vec{F}_t$ is bigger than $\vec{F}_g$, which makes sense since accelerating up.

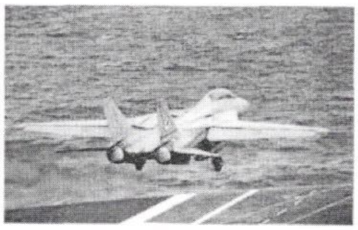
B: Sample Problems

Use your solution sheets to answer the following questions (pictorial, physics, word, mathematical representations)



1. Sunjamming. A "sun yacht" is a spacecraft with a large sail that is pushed by sunlight. Although such a push is tiny in everyday circumstances, it can be large enough to send the spacecraft outward from the Sun on a cost-free but slow trip. Your spacecraft has a mass of 900 kg and receives a steady push of 20 N from the sun. It starts its trip from rest. How far will it travel in 1.0 days and how fast will it then be moving?

2. Two People Pull. Two people are having a tug-of-war and pull on a 25 kg sled that starts at rest on frictionless ice. The forces suddenly change as one person tugs harder with a force of 92 N compared with the other person's force of 90 N. How quickly is the sled moving after 1.5 s?



3. Take Off. A Navy jet with a mass of 2.3×10^4 kg requires an airspeed of 85 m/s for liftoff. The engine develops a maximum force of 1.07×10^5 N, but that is insufficient for reaching takeoff speed in the 90 m runway available on an aircraft carrier. What minimum force (assumed constant) is needed from the catapult that is used to help launch the jet? Assume that the catapult and the jet's engine each exert a constant force over the 90 m distance used for takeoff.

Answers: (1) 8.29×10^7 m, 1.92×10^3 m/s, (2) 0.12 m/s, (3) 8.16×10^5 N

Adapted from Cummings, K., et al, *Understanding Physics*, Wiley, 2004

SPH3U: Freefall Acceleration

Last class we saw that an object in freefall accelerates at a constant rate when air resistance is not a large factor.

A: The Acceleration Experiment

We need to find out what the rate of acceleration is and what that rate might depend upon. We will perform an experiment, collect measurements and calculate the acceleration of an object under freefall. → drop from known height. → measure time. → calc. $\vec{v}_2$ → calc $\vec{a}$

In the absence of air resistance, all objects near the surface of the earth that fall freely, which means they experience no effects other than gravity, will accelerate at:

B: The Freefall Problem

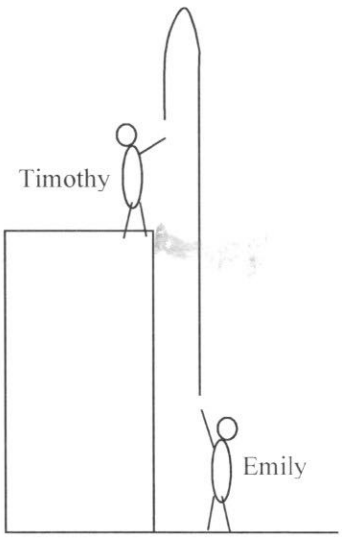
Timothy, a student no longer at our school, has very deviously hopped up on to the roof of the school. Emily is standing below and tosses a ball straight upwards to Timothy. It travels up past him, comes back down and he reaches out and catches it. Tim catches the ball 6.0 m above Emily's hands. The ball was travelling at 12.0 m/s upwards, the moment it left Emily's hand. We would like to know how much time this trip takes.

1. **Represent.** Complete part A below. Indicate the *y*-origin for position measurements and draw a sign convention where upwards is positive. Label the important events.

2. **Represent.** Complete part C below. Make sure the two graphs line-up vertically. Draw a single dotted vertical line through the graphs indicating the moment when the ball is at its highest.

A: Pictorial Representation

Sketch, coordinate system, label givens, conversions, describe events



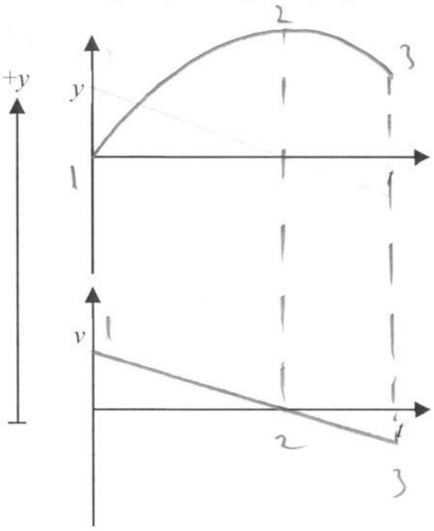
Event 1: ball leaves hand

Event 2: ball at top

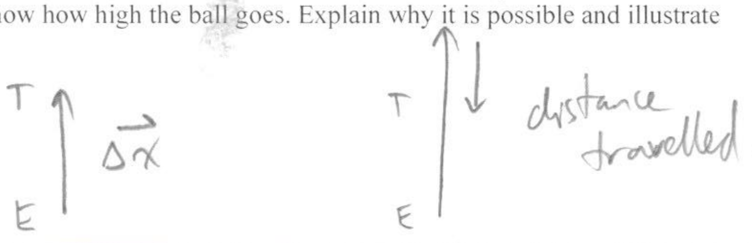
Event 3: ball arrives in hands

C: Physics Representation

Motion diagram, motion graphs, key events



3. **Reason.** We would like to find the total displacement of the ball while in freefall. Some students argue that we can't easily tell what the displacement is since we don't know how high the ball goes. Explain why it is possible and illustrate this displacement with an arrow on the sketch.



The total length of the path traveled by an object is the *distance*. The change in position, from one event to another is the *displacement*. Distance is a scalar quantity and displacement is a vector quantity.

4. **Reason.** The BIG 5 equations are valid for any interval of motion where the acceleration is uniform. Does the ball accelerate uniformly during events 1 and 3? Explain.

Yes it does – only affected by gravity

5. **Reason.** Isaac says, "I want to use an interval of time that ends when the ball comes to a stop in Tim's hand. Then we know that $v_2 = 0$." Why is Isaac incorrect? Explain.

We need to consider the moment just before it hits his hand... when it hits his hand, acceleration changes (not freefall)

6. **Solve.** Choose a BIG five equation to solve for the time. (Hint: one single BIG 5 equation will solve this problem). Note that you will need the quadratic formula! $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ You may leave out the units for the quadratic step.

7. **Solve**. Another method (method 2) would be to find v_3 first and then find the time.

D: Mathematical Representation

Describe steps, complete equations, substitutions with units, final statement

2 T $\vec{x} = 6.0$ m, $\vec{v}_2 = ?$, $\Delta t = ?$

$\vec{a} = -9.8$ m/s²

1 E $\vec{x} = 0$, $\vec{v}_1 = 12$ m/s

$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta x$

$\vec{v}_2^{\,2} = (12 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(6.0)$

$\vec{v}_2^{\,2} = 144 - 117.6$

$\vec{v}_2^{\,2} = 26.4$

$\vec{v}_2 = \sqrt{26.4}$

$\boxed{\vec{v}_2 = 5.1 \text{ m/s [down]}}$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$-5.1 \text{ m/s} = 12 \text{ m/s} + (-9.8 \text{ m/s}^2)\Delta t$

$-17.1 \text{ m/s} = (-9.8 \text{ m/s}^2)\Delta t$

$\Delta t = \frac{-17.1 \text{ m/s}}{-9.8}$ $\boxed{\Delta t = 1.7 \text{ s}}$

Homework: Freefalling

1. Isaac is practicing his volleyball skills by volleying a ball straight up and down, over and over again. His teammate Marie notices that after one volley, the ball rises 3.6 m above Isaac's hands. What is the speed with which the ball left Isaac's hand? (8.5 m/s)

2. With a terrific crack and the bases loaded, Albert hits a baseball directly upwards. The ball returns back down 4.1 s after the hit and is easily caught by the catcher, thus ending the ninth inning and Albert's chances to win the World Series. How high did the ball go? (21.0 m)

3. Emmy stands on a bridge and throws a rock at 7.5 m/s upwards. She throws a second identical rock with the same speed downwards. In each case, she releases the rock 10.3 m above a river that passes under the bridge. Which rock makes a bigger splash? (v_2=16.2 m/s)

The idea that the interaction forces between two objects (the carts) are equal in size holds true for all physical objects. This idea is known as ***Newton's 3rd Law***.

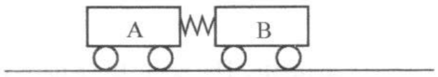
When objects interact, a pair of forces is always produced – they are two parts of one interaction. We call these two forces a *third law force pair.* The two forces that are members of the same third law pair share some important characteristics.

- the same magnitude
- opposite directions
- the same type (gravitational, normal, tension, etc.)
- start and stop acting at the same time
- act on different objects

We can use the *3rd law notation* for forces to help show these relationships. $\vec{F}_{n\ table-block}$ means the normal force of the table acting on the block.

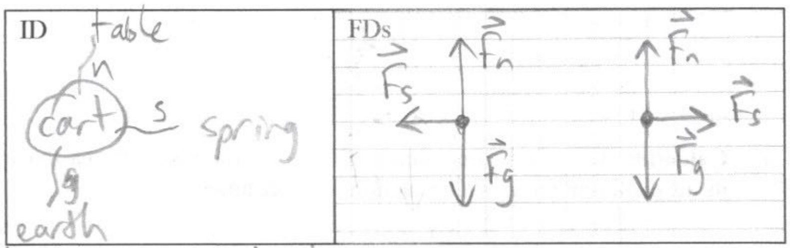
A: Exploding Carts!

Your teacher has two equal-mass dynamics carts on a track. Cart A has a compressed spring attached. Your teacher will release the spring and we will focus on the interval of time when the spring is expanding and affecting the carts.



1. **Represent.** Draw an ID for the two carts. Draw a FD for each cart.

2. **Predict.** Based on your diagrams, make a prediction comparing the velocity of each cart after the spring is released. Explain.

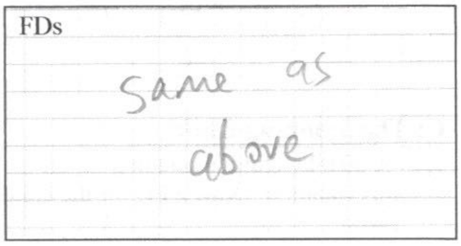


same – same force, same mass, ∴ same acceleration & velocity

3. **Test.** Observe the results. Do they agree with your predictions? yes!

4. **Predict and Test.** An extra mass is added to the cart with the spring. How do you think the FDs and resulting velocities change? Explain. You will test as a class.

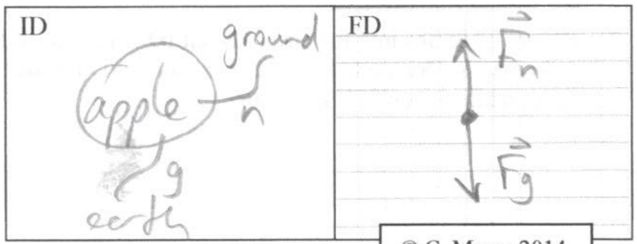
Same force, but cart with mass will have lower velocity



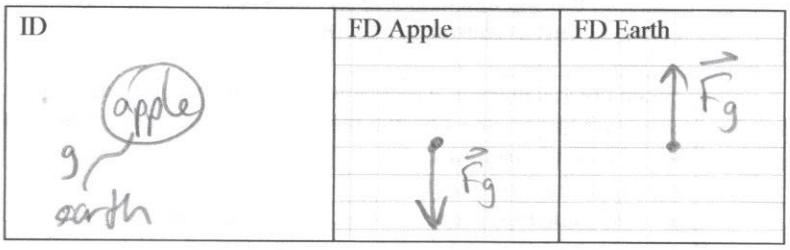
B: The Apple and the Earth

The story goes that our friend Sir Isaac Newton made a great discovery while he was sitting under an apple tree and an apple happened fall down on him.

1. **Represent.** Draw an ID and FD for the apple while it is at rest on the ground. Label each force using the 3rd law notation.



2. **Represent.** Draw an ID and two FDs for the apple and the earth *while the apple is falling*.



3. **Reason.** Marie says, "I think both the apple and the earth should be accelerating." Do you agree or disagree? Explain.

Agree - each has an unbalanced force.

4. **Reason.** Isaac says, "The earth clearly doesn't move, so I don't believe that it experiences an equal force to the apple." Do you agree or disagree? Explain. Disagree - the force is there, but the mass of the earth is so large that the force has almost no effect.

5. **Calculate.** The apple has a mass of 0.2 kg. What is the magnitude of the force of gravity it experiences? ($F_{net} = mg$)

$F_{net} = (0.2\ kg)(10\ m/s^2)$
$F_{net} = 2.0\ N$

6. **Calculate.** What is the magnitude of the force of gravity the earth experiences? If it has a mass of 6.0 x 10^{24} kg, what is its acceleration due to its interaction with the apple?

Also 2.0 N, but up.

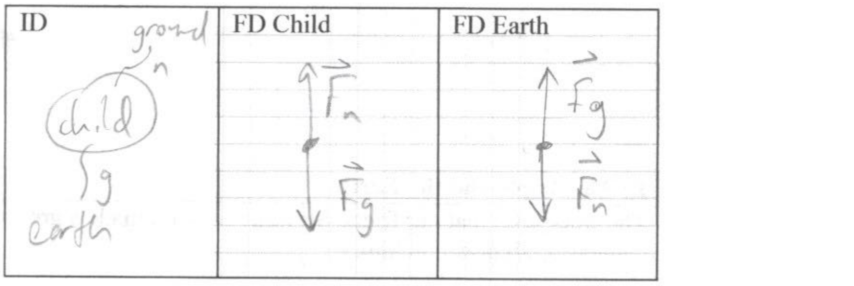
$2.0 = (6.0 \times 10^{24})(\vec{a})$
$\vec{a} = 3.3 \times 10^{-23}\ m/s^2$

7. **Explain.** The acceleration that the earth experiences due only to the apple is microscopic. What is another reason why things like apples in the air don't cause the earth any significant acceleration?

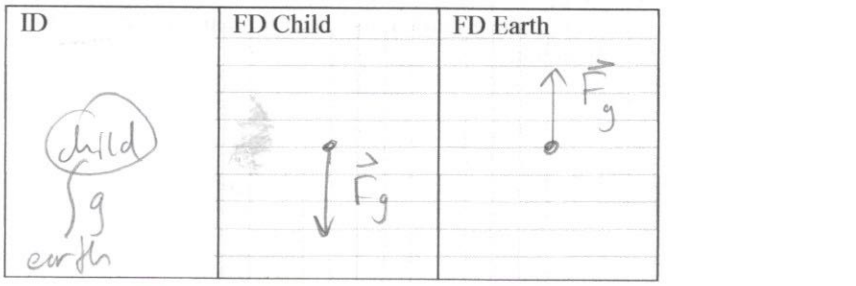
Probably things in the air on other side of the earth pulling the earth in the other direction.

C: The Jumping Child

1. A 29.0 kg child is standing on the floor. Draw an ID for this situation. Draw a FD for the child and a FD for the earth.



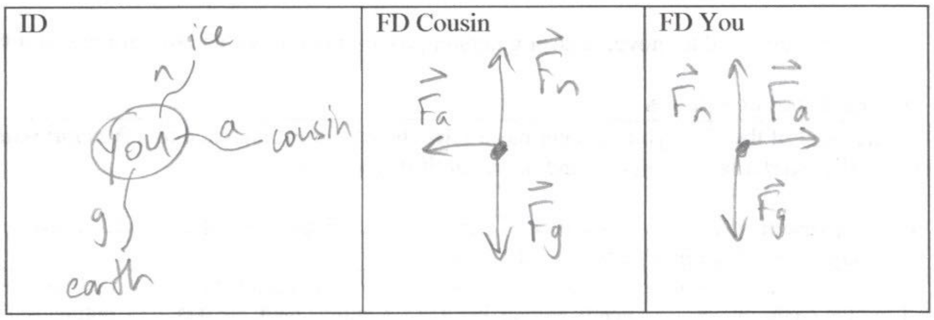
2. The child jumps into the air. Draw an ID while the child is in the air. Draw a FD for the child and a FD for the earth while the child is in the air.



A: Physics on Ice

You have brought your little cousin out skating for the very first time. Both of you are standing on the ice wearing skates (no friction) and are facing one another. Your little cousin is a bit timid and needs to hold on to your scarf while you pull.

1. **Represent.** Draw an ID that includes you and your cousin. Draw a FD for you and a FD for your cousin.



2. **Calculate.** Your cousin holds on while you gently pull the scarf with a 6 N force to start her moving. Her little mass is 17 kg. Determine her speed after pulling for 2.0 s.

$\vec{F}_{net} = m\vec{a}$ $\quad \vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$6\ N = (17\ kg)\vec{a}$ $\quad \vec{v}_2 = (0.35)(2.0)$

$\vec{a} = 0.35\ m/s^2$ $\quad \boxed{\vec{v}_2 = 0.71\ m/s}$

3. **Reason.** Albert says, "I understand why the cousin speeds up – you are pulling on the scarf and she holds on. But I don't predict you will move. Your cousin is only holding on, not pulling. And, in any case, she is much smaller so she couldn't pull you anyways." Do you agree or disagree? Explain.

If I pull her and she holds on, she is actually pulling me too.

4. **Represent and Calculate.** Use your actual mass to determine your speed after the same 2.0 seconds of pulling.

$6\ N = (85\ kg)\vec{a}$ $\quad \vec{v}_2 = (0.07)(2)$

$\vec{a} = \frac{6}{85} = 0.07\ m/s^2$ $\quad \vec{v}_2 = 0.14\ m/s$

B: Practice Problems

1.Two hockey players are standing on the ice. Player A has a mass of 100 kg, and Player B has a mass of 112 kg. One player pushes the other with a force of 50 N. Calculate the acceleration of each player. (ANS: $\vec{a_A} = -0.50\ \frac{m}{s^2}$ $\vec{a_B} = 0.45\ \frac{m}{s^2}$)

2. CHALLENGE!

~~Two boats~~ A and B, connected with a string, are floating on a calm lake. The mass of A and B are 100 kg and 150 kg, respectively, and the distance between them is 30 m. If the string is pulled from boat A with a constant force and the two boats meet in 20 seconds, what is **the distance that boat A moves?**

wording

3. Your friend is sitting on a skateboard. You stay in place on the road and give her a big push forwards to start her moving. The road is a bit bumpy, so the skateboard experiences some friction. You push on her with a 23 N force. The mass of your friend is 49 kg and the skateboard is 3.1 kg. While you push, your friend accelerates at 0.4 m/s². Find the size of **all** the horizontal forces in this situation. Draw a **single** interaction diagram showing two systems (you and your friend), and draw a force diagram for each system. Answers: A.2: 0.706 m/s, B: on you (23 N, 23 N), on your friend (23 N, 2.16 N)