$\frac{km/h}{s}$

SPH3U: Calculating Acceleration

$\vec{v} = \frac{\Delta x}{\Delta t} = \frac{\vec{x}_2 - \vec{x}_1}{\Delta t}$

A: Defining Acceleration

The expression $\Delta\vec{v}/\Delta t$ represents the *change* in velocity occurring in each unit of time and is called *acceleration* $\vec{a}$:

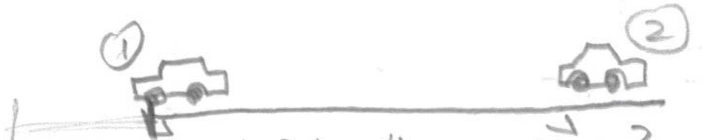
$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Note in the equation above, we wrote v_f and v_i for the ~~final and initial velocities~~ during some interval of time. If your time interval is defined by events 2 and 3, then v_3 is the final velocity and v_2 is the initial velocity.

B: Problem Solving

1. **Hit the Gas!** You are driving along the 401 and want to pass a large truck. You floor the gas pedal and begin to speed up. You start at 102 km/h, accelerate at a steady rate of 2.9 (km/h)/s (obviously not a sports car). What is your velocity after 5.3 seconds when you finally pass the truck?

A: Pictorial Representation
Sketch, coordinate system, label givens using symbols, describe events



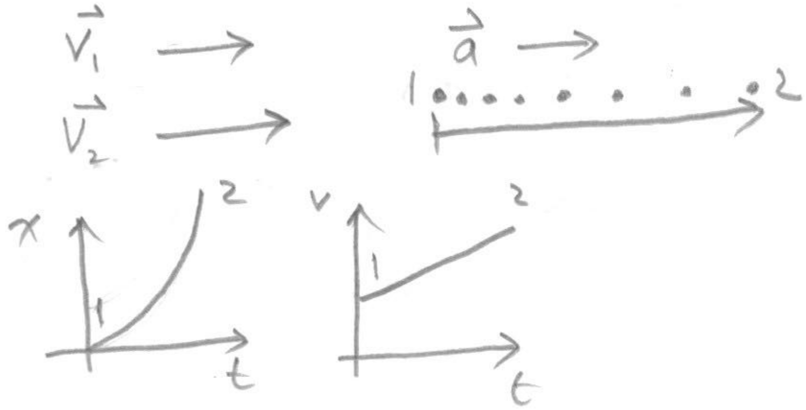
$\vec{v}_1 = 102$ km/h　　$\vec{v}_2 = ?$

$\Delta t = 5.3$ s

$\vec{a} = 2.9 \frac{km/h}{s}$

$\rightarrow +$

C: Physics Representation
Motion diagram, motion graphs, velocity vectors, events



B: Word Representation
Describe motion (no numbers), explain why, assumptions

- constant acceleration

D: Mathematical Representation
Number and describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\vec{a} = 2.9 \frac{km/h}{s}$

$\Delta t = 5.3$ s

$\vec{v}_i = 102$ km/h

$\vec{v}_f = ?$

$5.3 \cdot 2.9 = \frac{\vec{v}_f - 102}{\cancel{5.3}} \cancel{\times 5.3}$

$15.37 = \vec{v}_f - 102$

$+102 \qquad +102$

$117.37 = \vec{v}_f$ km/h

$\boxed{\vec{v}_f = 117 \text{ km/h}}$

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D: Mathematical Representation

Number and describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{v}_i + a\Delta t = \vec{v}_f$

$\vec{v}_i = 102$ km/h

$\vec{a} = 2.9 \frac{km/h}{s}$

$\Delta t = 5.3$ s

sub

$102 + 2.9(5.3) = \vec{v}_f$

$102 + 15.37 = \vec{v}_f$

$\boxed{117.4 \text{ km/h} = \vec{v}_f}$

4. **The Rocket** A rocket is travelling upwards. A second engine begins to fire causing it to speed up at a rate of 21 m/s^2. After 4.3 seconds it reaches a velocity of 413 km/h and the engine turns off. What was the velocity of the rocket when the second engine began to fire?

To describe motion in the vertical direction, use the symbol *y* for the *vertical* position. All other symbols remain the same. In physics, the symbol *x* will only be used for *horizontal* position. The sketch for the situation should show the vertical motion and the coordinate system should show which vertical direction is the +*y*-direction. The motion diagram and the velocity vectors should point vertically.

A: Pictorial Representation

Sketch, coordinate system, label givens & unknowns using symbols, conversions, describe events

(2) $\vec{v}_f = 413$ km/h

↑+

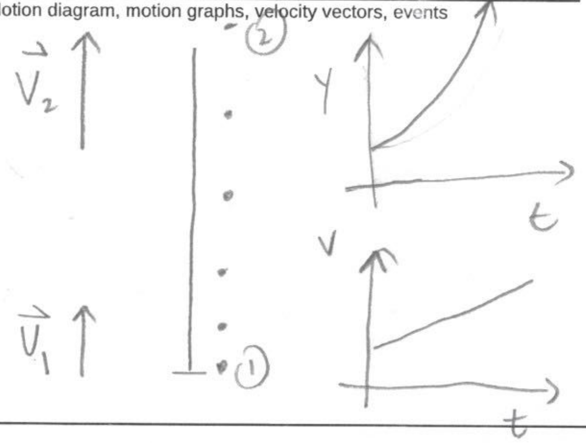
$\Delta t = 4.3$ s

$\vec{a} = 21$ m/s^2

(1) $\vec{v}_i = ?$

C: Physics Representation

Motion diagram, motion graphs, velocity vectors, events



B: Word Representation

Describe motion (no numbers), explain why, assumptions

Rocket travels upwards (+ direction), accelerates steadily from starting to ending velocity.

D: Mathematical Representation

Number and describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\vec{a} = 21$ m/s^2

$\vec{v}_i = ?$

$\vec{v}_f = 413$ km/s

$\Delta t = 4.3$ s

(1) Convert $\vec{a} = 21$ m/s^2 to $\frac{km/h}{s}$

$21 \frac{m}{s^2}\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 75.6 \frac{km/h}{s}$

(2) Substitute

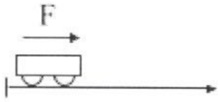
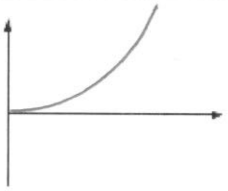
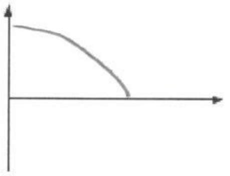
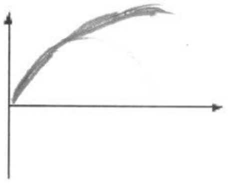
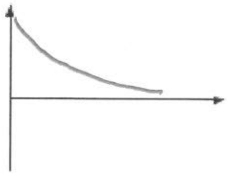
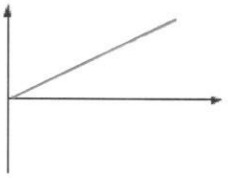
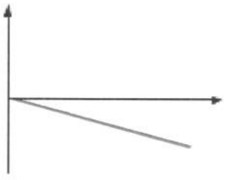
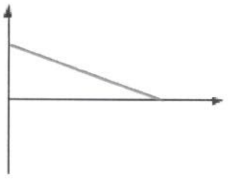
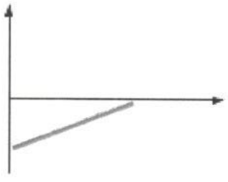
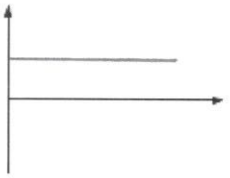
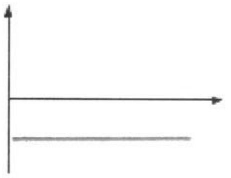
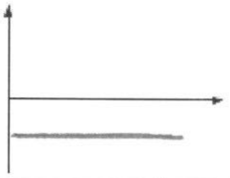
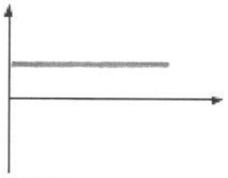
$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \Rightarrow 4.3 \times 75.6 = \frac{413 - \vec{v}_i}{4.3} \times 4.3$

$325.08 = 413 - \vec{v}_i$

$-87.92 = -\vec{v}_i$

$\boxed{\vec{v}_i \doteq 88 \text{ km/h}}$

→ +

	1	2	3	4
Description	The cart is released from rest near the motion detector. The fan pushes on the cart **away** from the detector.	The cart is released from rest far from the detector. The fan pushes **towards** the detector.	The cart is moving away from the detector. The fan pushes **towards** the detector.	The cart is moving towards the detector. The fan is pushing **away** from the detector.
Sketch with Force				
Position graph				
Velocity graph				
Acceleration graph				
Slowing down or speeding up?	speeding up	speeding up	slowing down	slowing down
Sign of Velocity	+	−	+	−
Sign of Acceleration	+	−	−	+

Acceleration is a **vector** quantity, so the sign indicates a direction. This is **not** the direction of the object's motion!

2. **Reason.** Emmy says, "We can see from these results that when the acceleration is positive, the object always speeds up." Do you agree with Emmy? Explain. No. In case 4, the cart has a positive acceleration but is slowing down.

3. **Reason.** What conditions for the acceleration and velocity must be true for an object to be speeding up? To be slowing down?
Signs must be same to be speeding up.
" " " opposite " " slowing down.

4. **Reason.** Which quantity in our chart above does the sign of the acceleration **always** match?
Direction of fan: away → positive acceleration towards → negative acceleration.

Always compare the magnitudes of the velocities, the speeds, using the terms *faster* or *slower*. Describe the motion of accelerating objects as *speeding up* or *slowing down* and state whether it is moving in the positive of negative direction. **Never** use the d-word, *deceleration* - yikes! Note that we will always assume the acceleration is uniform (constant).

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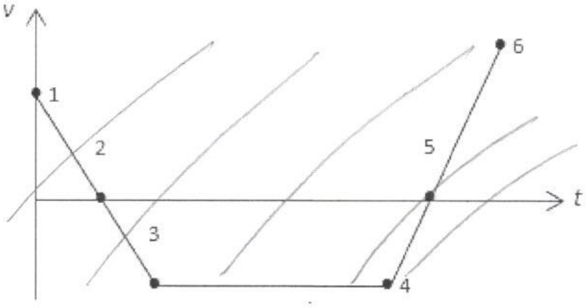
SPH3U Area and Displacement



A graph is more than just a line or a curve. We will discover a very handy new property of graphs which has been right under our noses (and graphs) all this time!

A: Looking Under the Graph

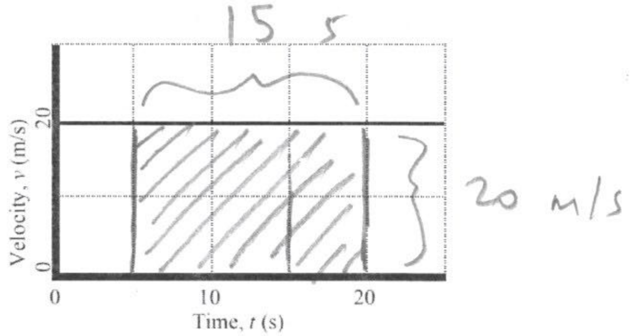
A car drives south along a straight road at 20 m/s. After 5 s the car passes a streetlight and at 20 s the car passes a bus stop.

1. **Calculate** the displacement of the car between the streetlight and the bus stop using the formula $\vec{v} = \Delta\vec{x}/\Delta t$

$\vec{V} = \frac{\Delta x}{\Delta t}$ $\quad 20 = \frac{\Delta x}{15}$ $\quad \boxed{\Delta x = 300\ m}$

2. **Sketch.** Now we will think about this calculation in a new way. Draw and shade a rectangle on the graph that fills in the area between the line of the graph and the time axis, for the time interval of 5 to 20 seconds.

3. **Interpret.** Calculate the area of the rectangle. Note that the length and width have a meaning in physics, so the final result is not a physical area. Use the proper physics units that correspond to the height and the width of the rectangle. What physics quantity does the final result represent?

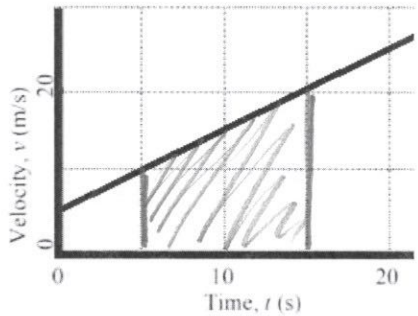


$(20\ \frac{m}{s})(15\ s) = 300\ m$

Area under a velocity graph. The *area* under a velocity-time graph for an interval of motion gives the *displacement* during that interval. Both velocity and displacement are vector quantities and can be positive or negative depending on their directions. According to our usual sign convention, areas above the time axis are positive and areas below the time axis are negative.

B: What if Velocity is not Constant?

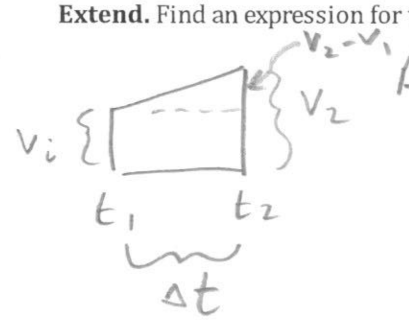
Consider the velocity time graph shown in the diagram.
Suppose we want to know how far the car travelled between $t_1 = 5.0$ s and $t_2 = 15.0$ s. Shade in the area representing the distance travelled over this interval.



Calculate the area of your shaded region. Remember to include units.

A = ▭ + ◺
$= (10)(10) + \frac{1}{2}(10)(10)$
$= 100 + 50$
A = 150 m/s

Extend. Find an expression for the area under the graph using some or all of the variables t_1, t_2, Δt, $\vec{v}_1$, $\vec{v}_2$



A = ▭ + ◺
$= (\Delta t)(\vec{V_1}) + \frac{1}{2}\Delta t(\vec{V_2} - \vec{V_1})$
$= \vec{V_1}\Delta t + \frac{1}{2}\vec{V_2}\Delta t - \frac{1}{2}\vec{V_1}\Delta t$
$= \frac{1}{2}\vec{V_1}\Delta t + \frac{1}{2}\vec{V_2}\Delta t$
$= \left(\frac{\vec{V_1} + \vec{V_2}}{2}\right)\Delta t$

SPH3U: The BIG Five

Last class we found three equations to help describe motion with constant acceleration. A bit more work along those lines would allow us to find two more equations which give us a complete set of equations for the five kinematic quantities.

A: The BIG Five – Revealed!

Here are the BIG five equations for uniformly accelerated motion.

The **BIG** Five	$\vec{v}_1$	$\vec{v}_2$	$\Delta\vec{x}$	$\vec{a}$	Δt
$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$	✓	✓		✓	✓
$\Delta\vec{x} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$	✓		✓	✓	✓
$\Delta\vec{x} = \vec{v}_2\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$		✓	✓	✓	✓
$\Delta\vec{x} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)\Delta t$	✓	✓	✓		✓
$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{x}$	✓	✓	✓	✓	

1. **Describe.** Define carefully each of the kinematic quantities in the chart below.

$\vec{v}_1$	initial velocity
$\vec{v}_2$	final velocity
$\Delta\vec{x}$	change in position (displacement)
$\vec{a}$	acceleration
Δt	change in time

2. **Reason.** What condition must hold true (we mentioned these in the previous investigation) in order to use the big 5 kinematic equations? acceleration must be constant

B: As Easy as 3-4-5

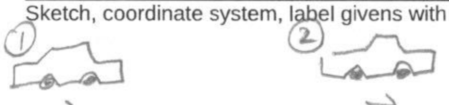
Solving a problem involving uniformly accelerated motion is as easy as 3-4-5. As soon as you know **three** quantities, you can always find a **fourth** using a BIG **five**! Write your solutions carefully using our solution process. Note that we are focusing on certain steps in our work here – in your homework, make sure you complete all the steps!

Problem 1

A traffic light turns green and an anxious student floors the gas pedal, causing the car to acceleration at 3.4 m/s² for a total of 10.0 seconds. We wonder: How far did the car travel in that time and what's the big rush anyways?

A: Pictorial Representation

Sketch, coordinate system, label givens with symbols, conversions, describe events



$\rightarrow +$

$\vec{v}_i = 0$ $\vec{v}_f = ?$

$\vec{a} = 3.4\ m/s^2$

$\Delta x = ?$

$\Delta t = 10\ s$

Emmy says, "I am given only two numbers, the acceleration and time. I need three to solve the problem. I'm stuck!" Explain how to help Emmy.

↳ stopped at start, so $\vec{v}_i = 0$

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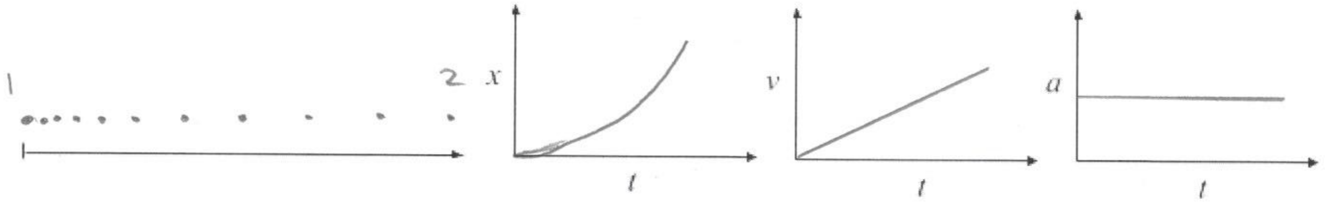
C: Physics Representation

Motion diagram, motion graphs, velocity vectors, events



D: Mathematical Representation

Describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{v_i} = 0$

$v_f = ?$

$\vec{\Delta x} = ?$

$\vec{a} = 3.4\ m/s^2$

$\Delta t = 10.0\ s$

① find final velocity

$\vec{v_2} = \vec{v_1} + \vec{a}\Delta t$

$\vec{v_2} = 0 + (3.4)(10.0)$

$\vec{v_2} = 34.0\ m/s$

② find distance travelled

$\Delta x = \vec{v_1}t + \frac{1}{2}\vec{a}(\Delta t)^2$

$\Delta x = (0)(10) + \frac{1}{2}(3.4)(10)^2$

$\boxed{\Delta x = 170\ m}$

$\therefore$ travelled 170 m

C: You've Got Problems: Complete these problems on a separate solution sheet

(1) **Crash Test.** An automobile safety laboratory performs crash tests of vehicles to ensure their safety in high-speed collisions. The engineers set up a head-on crash test for a Smart Car which collides with a solid barrier. The engineers know the car initially travels south at 100. km/h and the car crumples 0.780 m during the collision. The engineers have a couple of questions: How much time does the collision take? What was the car's acceleration during the collision? (0.0562 s, 495 m/s^2 [backwards])

(2) **Off the Wall.** An important part of Penny's swim race is when she turns around while pushing on the swimming pool wall. When she makes contact with the wall, she is travelling at 1.66 m/s east. After pushing against the wall for 0.30 s, she leaves contact with it and is travelling at 1.98 m/s west. What is her acceleration during this time? (12.1 m/s^2 west)

(3) **The Track.** A cart is placed at the bottom of an inclined track. It uses a spring to launch itself up the incline with a speed of 0.79 m/s. While travelling up and down the incline, the cart has an acceleration of 0.54 m/s^2. How much time does it take to make the complete trip up and back down to its starting position? (Hint: this is a one step problem) (2.9 s)

(4) **Taking Off.** A jumbo jet must reach a speed of 360 km/h on the runway for takeoff. What is the smallest constant acceleration needed to takeoff from a 1.80 km runway? Give your answer in m/s^2 (2.8 m/s^2)

(5) **Shuffleboard Disk.** A shuffleboard disk is accelerated at a constant rate from rest to a speed of 6.0 m/s over a 1.8 m distance by a player using a stick. The disk then loses contact with the stick and slows at a constant rate of 2.5 m/s^2 until it stops. What total distance does the disk travel? (Hint: how many events are there in this problem?) (9.0 m)