SPH3U: Algebra Review/Practice

A: Finding Unknown Quantities

In physics we will be dealing with various equations, and we will need to use algebra to find missing values.
There are two main methods that can be employed:

Method 1: Rearrange the equation to isolate the variable we want to find, then substitute and evaluate → this method is a little more abstract, but can save work in the long run since we only need to rearrange once	**Method 2**: Substitute the values of known variables, then rearrange and solve for the unknown → this method is less abstract, but is more work in the long run since we will need to rearrange for every question

Given the equation $A = \frac{a+b}{2} \cdot h$, find a if $A = 20$ cm^2, $b = 6$ cm, $h = 4$ cm

Method 1

$A = \frac{a+b}{2} \cdot h$

$\frac{A}{h} = \frac{a+b}{2}$

$\frac{2A}{h} = a+b$

$\frac{2A}{h} - b = a$

$a = \frac{2(20)}{4} - 6$

$a = 4$ cm

Method 2

$A = \frac{a+b}{2} \cdot h$

$20 = \frac{a+6}{2} \cdot 4$

$20 = 2(a+6)$

$10 = a+6$

$4 = a$

$a = 4$ cm

B: Practice

Find the missing value. In general you can use either method, although it's desirable to be able to use either.

1. Given $P = 20$, $l = 8$
EQN: $P = 2l + 2w$ Find w

$P - 2l = 2w$

$\frac{P-2l}{2} = w$

$\frac{20-2(8)}{2} = w$

$2 = w$

2. Given $y = 5$, $m = 10$, $b = 3$
EQN: $y = mx + b$ Find x

$5 = 10x + 3$

$-3 \quad -3$

$2 = 10x$

$\frac{2}{10} = x$

$0.2 = x$

3. Given $\vec{v}_2 = 30$, $\vec{v}_1 = 0.5$, $\vec{a} = 6$
EQN: $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$ Find Δt

$\vec{v}_2 - \vec{v}_1 = \vec{a}\Delta t$

$\frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \Delta t$

$\frac{30-0.5}{0.6} = \Delta t$

$\Delta t = 49.2$

4. Given $\vec{v}_2 = 15$ $\vec{v}_{avg} = 26$
Find $\vec{v}_1$ EQN: $\vec{v}_{avg} = \frac{\vec{v}_1 + \vec{v}_2}{2}$

$26 = \frac{\vec{v}_1 + 15}{2}$

$52 = \vec{v}_1 + 15$

$-15 \quad -15$

$37 = \vec{v}_1$

5. Given $\vec{a} = 0.5$, $\vec{v}_f = 30$, $\Delta t = 6$
Find $\vec{v}_i$ EQN: $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\vec{a}\Delta t = \vec{V}_F - \vec{V}_i$

$\vec{a}\Delta t - \vec{V}_F = -\vec{V}_i$

$\vec{V}_i = \vec{V}_F - \vec{a}\Delta t$

$\vec{V}_i = 30 - (0.5)(6)$

$\vec{V}_i = 27$

6. Given $\vec{v}_2 = 8$, $\vec{v}_1 = 5$, $\vec{a} = 1.5$
Find $\Delta\vec{x}$ EQN: $\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{x}$

$8^2 = 5^2 + 2(1.5)\Delta\vec{x}$

$64 = 25 + 3\Delta\vec{x}$

$39 = 3\Delta\vec{x}$

$13 = \Delta\vec{x}$

SPH3U: Calculating Acceleration

A: Defining Acceleration

The expression $\Delta\vec{v}/\Delta t$ represents the *change* in velocity occurring in each unit of time and is called *acceleration* $\vec{a}$:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Note in the equation above, we wrote v_f and v_i for the final and initial velocities during some interval of time. If your time interval is defined by events 2 and 3, then v_3 is the final velocity and v_2 is the initial velocity.

B: A Few Problems!

Solve the first problem on your whiteboard, then do the rest of the problems on your own.

1. A car is speeding up with constant acceleration. You have a radar gun and stopwatch. You first notice the car moving at a velocity of 4.6 m/s and then 90.0 s later it is moving with a velocity of 8.2 m/s. What is the car's acceleration?

$$\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t}$$

$$= \frac{8.2 - 4.6}{90}$$

$$= \frac{3.6}{90}$$

$$= 0.04 \text{ m/s}^2$$

2. **Explain.** Show the algebraic steps required to isolate for $\vec{v}_f$ in the equation:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\Delta t \times \vec{a} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t} \times \Delta t$$

$$a\Delta t = \vec{V}_f - \vec{V}_i$$

$$+\vec{V}_i \qquad +\vec{V}_i$$

$$\boxed{\vec{V}_i + a\Delta t = \vec{V}_f}$$

3. **Hit the Gas!** You are driving along the 401 and want to pass a large truck. You floor the gas pedal and begin to speed up. You start at 102 km/h, accelerate at a steady rate of 2.9 (km/h)/s (obviously not a sports car). What is your velocity after 5.3 seconds when you finally pass the truck?

A: Pictorial Representation

Sketch, coordinate system, label givens using symbols, describe events

$\vec{V}_i = 102$ km/h

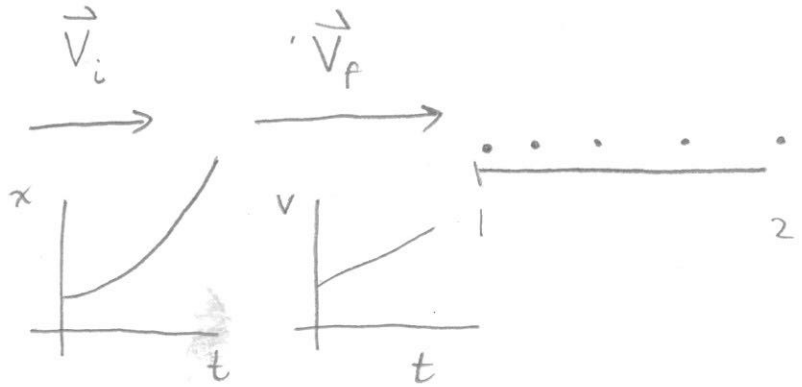
$\vec{V}_f = ?$

$\Delta t = 5.3$ s

$\vec{a} = 2.9 \frac{\text{km/h}}{\text{s}}$

C: Physics Representation

Motion diagram, motion graphs, velocity vectors, events



B: Word Representation

Describe motion (no numbers), explain why, assumptions

starts at $\vec{V}_i$, accelerates steadily to $\vec{V}_f$ over a period of time

D: Mathematical Representation

Number and describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{V}_i + a\Delta t = \vec{V}_f$

$102 + 2.9(5.3) = \vec{V}_f$ ← sub

$102 + 15.37 = \vec{V}_f$

$117.4 \text{ km/h} = \vec{V}_f$

$\vec{V}_i = 102 \text{ km/h}$

$\vec{a} = 2.9 \text{ km/h/s}$

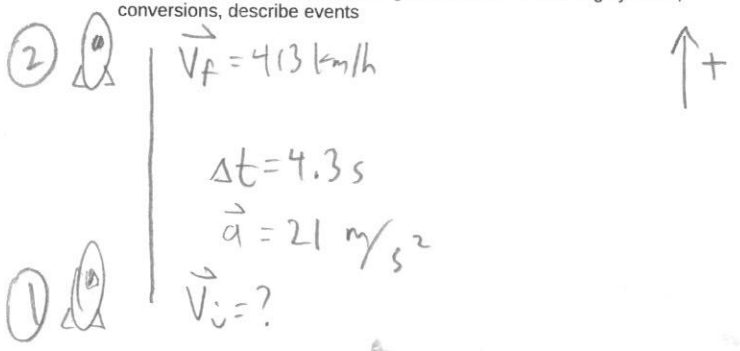
$\Delta t = 5.3 \text{ s}$

4. **The Rocket** A rocket is travelling upwards. A second engine begins to fire causing it to speed up at a rate of 21 m/s^2. After 4.3 seconds it reaches a velocity of 413 km/h and the engine turns off. What was the velocity of the rocket when the second engine began to fire?

To describe motion in the vertical direction, use the symbol *y* for the *vertical* position. All other symbols remain the same. In physics, the symbol *x* will only be used for *horizontal* position. The sketch for the situation should show the vertical motion and the coordinate system should show which vertical direction is the +*y*-direction. The motion diagram and the velocity vectors should point vertically.

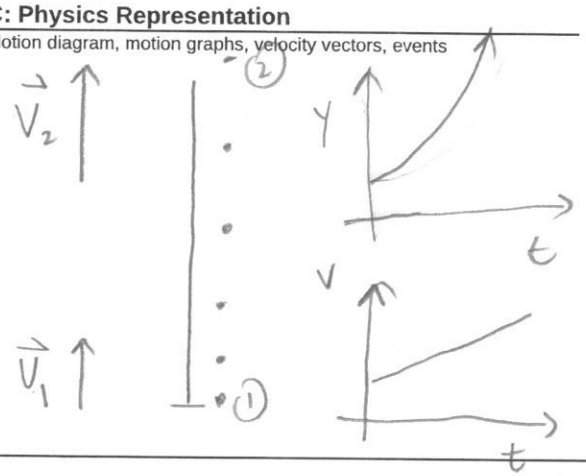
A: Pictorial Representation

Sketch, coordinate system, label givens & unknowns using symbols, conversions, describe events



C: Physics Representation

Motion diagram, motion graphs, velocity vectors, events



B: Word Representation

Describe motion (no numbers), explain why, assumptions

Rocket travels upwards (+ direction), accelerates steadily from starting to ending velocity.

D: Mathematical Representation

Number and describe steps, complete equations, algebraically isolate, substitutions with units, final statement

$\vec{a} = \dfrac{\vec{V}_f - \vec{V}_i}{\Delta t}$

$\vec{a} = 21 \text{ m/s}^2$

$\vec{V}_i = ?$

$\vec{V}_f = 413 \text{ km/s}$

$\Delta t = 4.3 \text{ s}$

① Convert $\vec{a} = 21 \text{ m/s}^2$ to km/h/s

$$21 \frac{\text{m}}{\text{s}^2}\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 75.6 \frac{\text{km/h}}{\text{s}}$$

② Substitute $\vec{a} = \dfrac{\vec{V}_f - \vec{V}_i}{\Delta t} \Rightarrow 4.3 \times 75.6 = \dfrac{413 - \vec{V}_i}{4.3} \times 4.3$

$325.08 = 413 - \vec{V}_i$

$-87.92 = -\vec{V}_i$

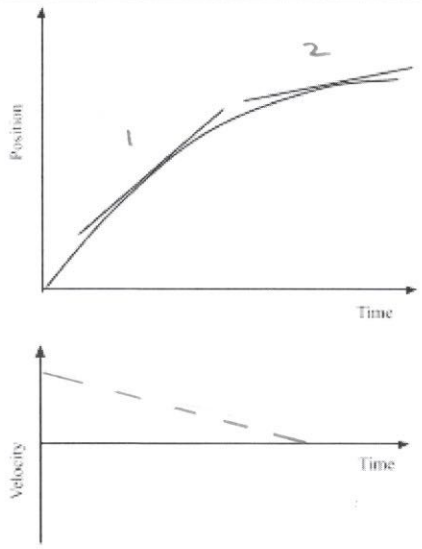
$\vec{V}_i = 88 \text{ km/h}$

SPH3U: Speeding Up or Slowing Down?

There is one mystery concerning acceleration remaining to be solved. Our definition of acceleration, $\Delta v/\Delta t$, allows the result to be either positive or negative, but what does that mean? Today we will get to the bottom of this.

A: Acceleration in Graphs

Your teacher has set-up a cart with a fan on a dynamics track and a motion detector to help create position-time and velocity-time graphs. Let's begin with a position graph before we observe the motion. The cart is initially moving forward. The fan is on and gives the cart a steady, gentle push which causes the cart to accelerate.



1. **Interpret.** What does the slope of a tangent to a position-time graph represent?

   instantaneous velocity

2. **Predict.** What will the velocity-time graph look like? Use a dashed line to sketch this graph. Compare your answer with your group and draw your velocity-time graph on a whiteboard.

3. **Interpret.** Is the cart speeding up or slowing down? Use the two tangents to the graph to help explain.

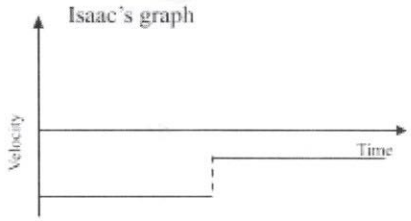
   Slowing down – at point 1 the slope (speed) is greater than at point 2

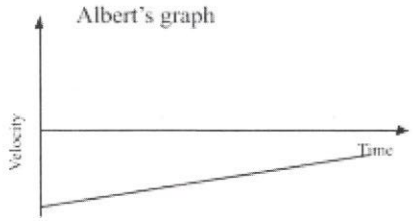
To help interpret position graphs, we will use the *tangent trick*. Use a ruler or pencil as the tangent line to a position graph. Interpret the slope of the tangent. Then move the tangent to a new spot along the graph and interpret. Decide if the object is speeding up or slowing down. This trick can also be used decide how to sketch a position graph.

4. **Reason.** Is the change in velocity positive or negative? What does this tell us about the acceleration?

   Negative – tells us it is decelerating

5. **Reason.** Two students draw a velocity graph based on the position graph above. Which graph do you think best matches the position graph? Explain.





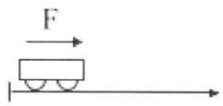
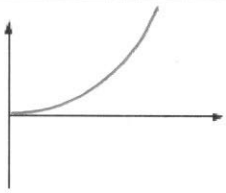
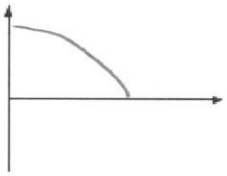
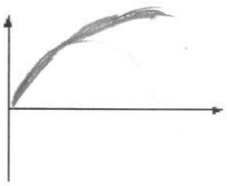
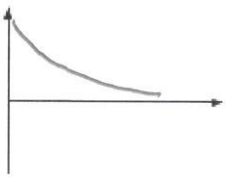
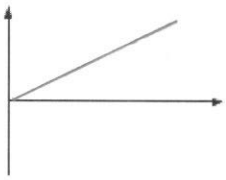
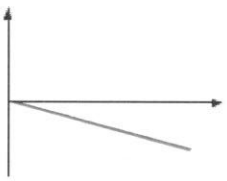
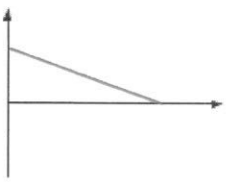
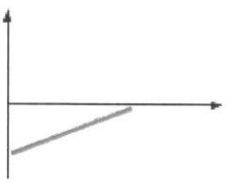
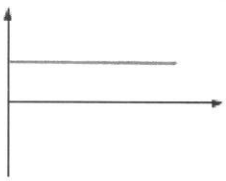
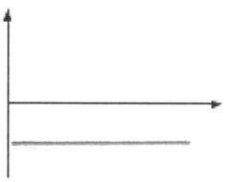
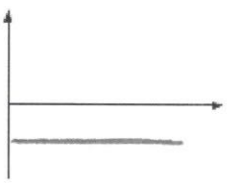
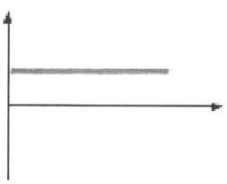
Albert: velocity is changing steadily, not suddenly.

B: The Sign of the Acceleration

Your teacher has a cart with a fan set up on a track.

1. **Observe, Predict and Interpret.** (*as a class)* Your teacher will lead you through four different situations involving the cart. You will make observations, make prediction and interpret the results using the chart on the next page.

→ +

	1	2	3	4
Description	The cart is released from rest near the motion detector. The fan pushes on the cart **away** from the detector.	The cart is released from rest far from the detector. The fan pushes **towards** the detector.	The cart is moving away from the detector. The fan pushes **towards** the detector.	The cart is moving towards the detector. The fan is pushing **away** from the detector.
Sketch with Force				
Position graph				
Velocity graph				
Acceleration graph				
Slowing down or speeding up?	speeding up	speeding up	slowing down	slowing down
Sign of Velocity	+	−	+	−
Sign of Acceleration	+	−	−	+

Acceleration is a **vector** quantity, so the sign indicates a direction. This is **not** the direction of the object's motion!

2. **Reason.** Emmy says, "We can see from these results that when the acceleration is positive, the object always speeds up." Do you agree with Emmy? Explain.

No. In case 4, the cart has a positive acceleration but is slowing down.

3. **Reason.** What conditions for the acceleration and velocity must be true for an object to be speeding up? To be slowing down?

Signs must be same to be speeding up.
" " " opposite " " slowing down.

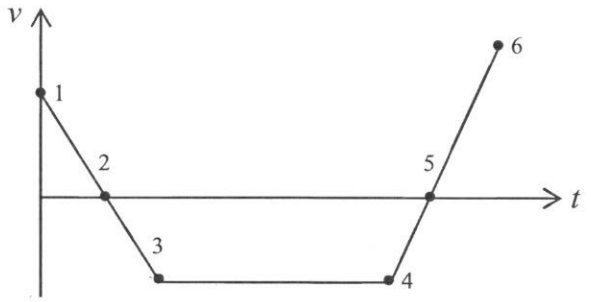
4. **Reason.** Which quantity in our chart above does the sign of the acceleration **always** match?

Direction of fan: away → positive acceleration towards → negative acceleration.

Always compare the magnitudes of the velocities, the speeds, using the terms *faster* or *slower*. Describe the motion of accelerating objects as *speeding up* or *slowing down* and state whether it is moving in the positive of negative direction. **Never** use the d-word, *deceleration* - yikes! Note that we will always assume the acceleration is uniform (constant).

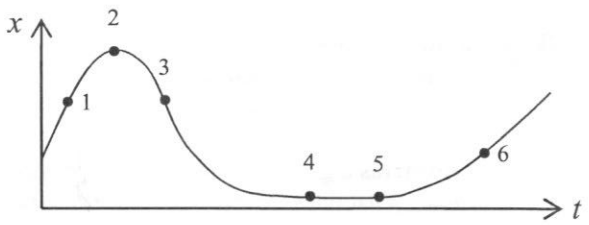
Homework: Speeding Up and Slowing Down

1. **Interpret and Explain.** A person walks back and forth in front of a motion detector producing the velocity graph shown to the right. Six events have been labelled on the graph. The chart below lists different examples of motion. Find the appropriate interval(s) of time in the graph that correspond to that type of motion and **provide evidence from the graph** supporting your choice.



Type of motion	Interval(s)	Evidence
positive acceleration	4-5 5-6	velocity is increasing
negative acceleration and a positive velocity	1-2	velocity is positive but decreasing ↳ negative acceleration
acceleration of zero	3-4	velocity is constant
speeding up	2-3 5-6	→ velocity is negative, as is acceleration → velocity is positive, as is acceleration
slowing down	1-2 4-5	→ velocity is positive but decreasing → velocity is negative but increasing
at rest (reminder: at rest means not moving for an **interval** of time)	None	no periods of time with $\vec{v}=0$
Change of acceleration	Moments: 3, 4	3 → negative acceleration stops 4 → positive acceleration begins

2. **Interpret and Explain.** In a different experiment, a person walks back and forth in front of a motion detector and produces the position graph shown to the right. The chart below lists different examples of motion. Find the appropriate interval(s) of time or events in the graph that correspond to that type of motion and **provide evidence from the graph** supporting your choice.



Type of motion	Intervals or Events	Evidence
Zero velocity	4-5	position is not changing
Speeding up	2-3 5-6	→ getting steeper towards sensor → curve getting steeper (moving faster away)
Slowing down	1-2 3-4	→ moving away but levelling off → moving towards but levelling off
Turning around	2	changes from x increasing to decreasing
	4-5	→ long pause but resumes in opposite direction